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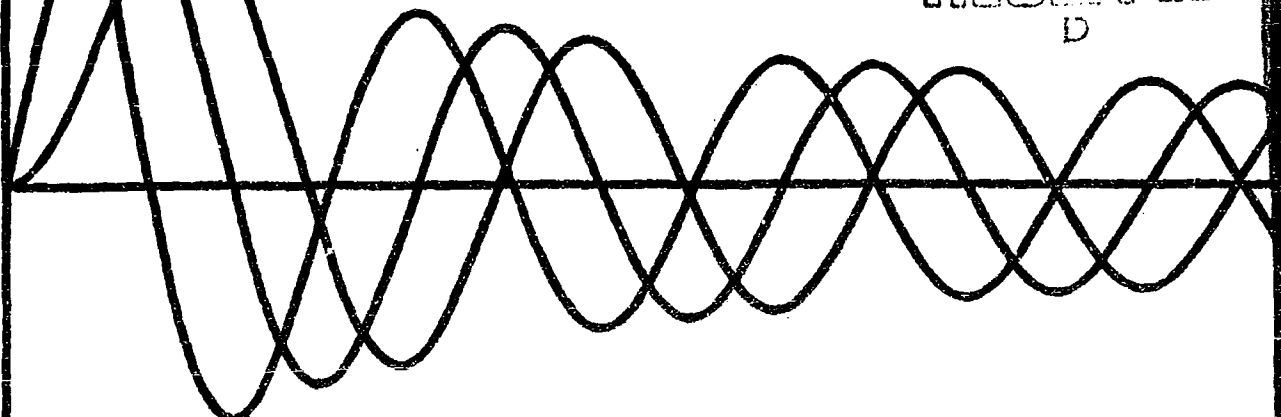
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Optimization of the reradiation
pattern of a Van Atta reflector

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ABSTRACT

An optimization of the reradiated field from a linear Van Atta reflector consisting of four equispaced parallel half-wave dipoles is made.

Before the optimization process is discussed, a further and more extensive theoretical investigation than the one given in Scientific Report no. 2 is made. A general expression for the reradiated field is established so that it is possible to study the influence of asymmetries in the location of the dipoles, unequal length of the transmission lines, and a mismatch between the dipoles and the transmission lines. In particular, the dependence of the reradiation pattern on the length and the characteristic impedance of the transmission lines, the imaginary part of the dipole impedances, and the spacing between the dipoles, is investigated.

The reradiation pattern is optimized so that the back-scattering, as a function of the angle of incidence, has its largest minimum value. The optimization is performed both when coupling is neglected and when it is taken into account. In the optimization consideration is given to the shape of the reradiation pattern near the direction back in the direction of propagation of an incident wave.

1. INTRODUCTION

The Van Atta array is assumed to operate in the following manner: an incident plane wave generates currents in the antennas composing the array in such a way that their fields are in phase back in the direction of incidence, for all angles of incidence. If this is the case, we say that the reflector has a Van Atta effect and it will always have a maximum of reradiation back in the direction of incidence. However, the theory applied to a Van Atta array consisting of half-wave dipoles (1, 2) shows that this is only true to a limited extent. In order to make a further study of this fact a method is presented for computing a figure which may be used as a measure of the amount by which the fields are out of phase in the direction considered. Since, in the computation, due attention is given to all angles of incidence, we say that the figure is a measure of the deviation from Van Atta effect. Therefore, the figure computed will be referred to as the deviation from Van Atta effect.

For convenience let us at this point introduce some concepts which were used in the previous reports and which also will be used here to describe the behaviour of the reflector. In Fig. 1 is shown the four dipoles. The direction back in the direction of propagation of an incident plane wave is called the Van Atta direction. Another direction of interest is the mirror direction or specular direction which would be the direction of the reflected principal ray if the dipoles were replaced by a metallic plate parallel to the plane through their axes. The direction of incidence is in the plane normal to the axes of the dipoles and we will look at the reradiation in this plane. The incident plane wave is polarized parallel to the dipoles. By the back-scattering (back-reradiation) we mean the reradiation in the Van Atta direction.

The following results obtained in Scientific Report no. 2 (2) are of interest in this report:

1. Only to some extent the reflector has a Van Atta effect.
2. The reflector has a mirror effect (specular effect) to the same degree as it has a Van Atta effect. (When we say that a reflector has a mirror effect, we mean that the fields from the currents in the dipoles are in phase in the mirror direction for all angles of incidence).
3. The coupling may decrease the deviation from the Van Atta and/or the mirror effect.
4. The deviation from Van Atta effect depends on the length of the transmission lines.

In this report we shall consider the dependence of the deviation from the Van Atta effect not only upon the length of lines but also upon spacing and matching between dipoles and interconnecting transmission lines. The matching is varied by changing the characteristic impedance of the transmission lines and the imaginary part of the antenna impedance.

In the following the length and characteristic impedance of the lines, the spacing, and the imaginary part of the antenna impedance will be referred to as the parameters of the reflector.

The coupling is taken into account by using the values of the mutual impedances calculated for two half-wave dipoles in free space. The fact that the mutual impedance of any two dipoles depends on the presence of the remaining dipoles is neglected. A simple method which takes this into account does not seem to exist. However, it is assumed that this approximation does not cause significant error.

The theoretical and numerical investigation is made for four dipoles only. The four dipoles may be considered as two pairs of antennas in an arbitrary Van Atta reflector and, because of this, it is possible to derive similar results for reflectors with more than four elements. However, the computer time used for investigating the simple reflector is rather large, and will increase rapidly when more elements are taken into account.

2. THE RERADIATION PATTERN AND THE VAN ATTA EFFECT

For the reflector described in Section 2.1, the expression for the reradiation pattern will be derived in Section 2.2. In Section 2.3 the condition for Van Atta effect will be derived and in Section 2.4 a measure of the deviation from Van Atta effect will be established. In Section 2.5 we find the circumstances in which the back-scattering, as a function of the angle of incidence, has its largest minimum value. Finally, the specular reflection will be considered in Section 2.6.

2.1. The reflector

For a linear Van Atta array consisting of four equispaced parallel half-wave dipoles we shall derive an expression for the reradiated field from which it is possible to examine the influence of asymmetries in location of the dipoles, unequal lengths of the transmission lines, and a mismatch between the dipoles and the transmission lines. In order to do this we will consider four dipoles the centers of which are placed arbitrarily with respect to each other in a plane. The dipoles are perpendicular to this plane (see Fig. 2). Furthermore, they are connected in pairs by transmission lines whose lengths need not be equal. It is assumed that the dipoles are numbered from 1 to 4 and that dipole 1 is connected to dipole 4 and dipole 2 to dipole 3. The degree of mismatch between the dipoles and the transmission lines is altered by changing the characteristic impedances of the transmission lines and the reactances of the dipoles. During the investigation the characteristic impedances of the two lines are kept equal. The half-wave dipole reactances are changed by the same amount by means of impedance transforming networks.

Most of the equations given in this chapter are valid for the general geometrical configuration of dipoles described above. However, attention is often given to the case for which the centers of the dipoles in each pair are placed symmetrically (not always on a line) about a common point.

2.2. The reradiated field

The equivalent circuit for dipoles 1 and 4 is shown in Fig. 3. A T-circuit with impedances Z_{B1} and Z_{C1} is used as an equivalent circuit for the transmission line of length l_1 connecting dipoles 1 and 4. For the transmission line of length l_2 connecting dipoles 2 and 3, a T-circuit with impedances Z_{B2} and Z_{C2}

is used. V_1, V_2, V_3 , and V_4 are the free-space open-circuit voltages induced by an incident field in dipoles 1, 2, 3, and 4, respectively. I_1, I_2, I_3 , and I_4 are the currents in the four dipoles composing the reflector. The mutual impedance between dipoles r and s ($r, s = 1, 2, 3, 4$) is denoted by Z_{rs} ($r \neq s$).

From the two equivalent circuits is readily obtained

$$(Z_{An} + Z_{B1} + Z_{C1})I_1 + Z_{12}I_2 + Z_{13}I_3 + (Z_{C1} + Z_{14})I_4 = V_1 \quad (1)$$

$$Z_{12}I_1 + (Z_{An} + Z_{B2} + Z_{C2})I_2 + (Z_{C2} + Z_{23})I_3 + Z_{24}I_4 = V_2 \quad (2)$$

$$Z_{13}I_1 + (Z_{C2} + Z_{23})I_2 + (Z_{An} + Z_{B2} + Z_{C2})I_3 + Z_{34}I_4 = V_3 \quad (3)$$

$$(Z_{C1} + Z_{14})I_1 + Z_{24}I_2 + Z_{34}I_3 + (Z_{An} + Z_{B1} + Z_{C1})I_4 = V_4 \quad (4)$$

By subtracting Equation (4) multiplied by $e^{i\beta l_1}$ from Equation (1) is found

$$\begin{aligned} & (Z_{An} + Z_{B1} + (1 - e^{i\beta l_1})Z_{C1} - Z_{14}e^{i\beta l_1})I_1 \\ & + (Z_{12} - Z_{24}e^{i\beta l_1})I_2 \\ & + (Z_{13} - Z_{34}e^{i\beta l_1})I_3 \\ & + ((-Z_{An} - Z_{B1} - Z_{C1}(1 - e^{-i\beta l_1}))e^{i\beta l_1} + Z_{14})I_4 = V_1 - V_4e^{i\beta l_1}. \end{aligned} \quad (5)$$

From this equation is obtained

$$\begin{aligned} & (Z_{An} + Z_0 - Z_{14}e^{i\beta l_1})I_1 + (Z_{12} - Z_{24}e^{i\beta l_1})I_2 + (Z_{13} - Z_{34}e^{i\beta l_1})I_3 + ((-Z_{An} + Z_0)e^{i\beta l_1} + Z_{14})I_4 \\ & = V_1 - V_4e^{i\beta l_1}, \end{aligned} \quad (6)$$

by using the relations

$$Z_{B1} + (1 - e^{i\beta l_1})Z_{C1} = Z_0 \quad (7)$$

$$-Z_{B1} - (1 - e^{-i\beta l_1})Z_{C1} = Z_0 \quad (8)$$

which may be derived from the expressions for the impedances of a T-circuit equivalent to a transmission line of length ℓ_1 :

$$Z_{B1} = -iZ_0 \operatorname{tg} \left(\frac{\beta \ell_1}{2} \right) \quad (9)$$

$$Z_{C1} = \frac{iZ_0}{\sin(\beta \ell_1)} \quad (10)$$

where Z_0 is the characteristic impedance of the transmission line.

From Equations (1) - (4) we obtain in the manner shown above the following system of equations determining the currents in the dipoles

$$\begin{aligned} (Z_{An} + Z_0 - Z_{14} e^{i\beta \ell_1}) I_1 + (Z_{12} - Z_{24} e^{i\beta \ell_1}) I_2 + (Z_{13} - Z_{34} e^{i\beta \ell_1}) I_3 + ((-Z_{An} + Z_0) e^{i\beta \ell_1} + Z_{14}) I_4 \\ = V_1 - V_4 e^{i\beta \ell_1} \end{aligned} \quad (11)$$

$$\begin{aligned} (Z_{12} - Z_{13} e^{i\beta \ell_2}) I_1 + (Z_{An} + Z_0 - Z_{23} e^{i\beta \ell_2}) I_2 + ((-Z_{An} + Z_0) e^{i\beta \ell_2} + Z_{23}) I_3 + (Z_{24} - Z_{34} e^{i\beta \ell_2}) I_4 \\ = V_2 - V_3 e^{i\beta \ell_2} \end{aligned} \quad (12)$$

$$\begin{aligned} (Z_{13} - Z_{12} e^{i\beta \ell_2}) I_1 + ((-Z_{An} + Z_0) e^{i\beta \ell_2} + Z_{23}) I_2 + (Z_{An} + Z_0 - Z_{23} e^{i\beta \ell_2}) I_3 + (Z_{34} - Z_{24} e^{i\beta \ell_2}) I_4 \\ = V_3 - V_2 e^{i\beta \ell_2} \end{aligned} \quad (13)$$

$$\begin{aligned} ((-Z_{An} + Z_0) e^{i\beta \ell_1} + Z_{14}) I_1 + (Z_{24} - Z_{12} e^{i\beta \ell_1}) I_2 + (Z_{34} - Z_{13} e^{i\beta \ell_1}) I_3 + (Z_{An} + Z_0 - Z_{14} e^{i\beta \ell_1}) I_4 \\ = V_4 - V_1 e^{i\beta \ell_1} \end{aligned} \quad (14)$$

The transformation of Equations (1) - (4) to Equations (11) - (14) is made in order to obtain a system of equations which is valid for all values of ℓ_1 and ℓ_2 . This is not the case for Equations (1) - (4) (see Equations (9) and (10) for Z_{B1} and Z_{C1}).

Equations (11) - (14) reduce to Equations (1) - (4) of Scientific Report no. 2, when $l_1 = l_2 = l$, the dipoles equispaced on a line, and $Z_{An} = R_{An} = Z_0$.

Equations (11) - (14) may be generalized in the following manner so as to be valid for a Van Atta array consisting of n pairs of dipoles. Let $z_{r,s}$ be the element in the r 'th row and the s 'th column in a square matrix $\{z\}$ of order $2n$. Then the matrix equation for n pairs of dipoles is

$$\{z\}\{i\} = \{v\}, \quad (15)$$

where

$$z_{r,s} = Z_{r,s} - Z_{2n-r+1,s} \cdot e^{i\beta l}, \quad (16)$$

except for $s = r$ and $s = 2n-r+1$ (diagonal elements). In these two cases we have

$$z_{r,r} = Z_{An} + Z_0 - Z_{2n-r+1,r} \cdot e^{i\beta l} \quad (17)$$

and

$$z_{r,2n-r+1} = (-Z_{An} + Z_0)e^{i\beta l} + Z_{r,2n-r+1}, \quad (18)$$

respectively. Here, $Z_{r,s} = Z_{s,r}$ is the mutual impedance between dipoles r and s ($r \neq s$). $Z_{r,s}$ corresponds to Z_{rs} given above. $\{i\}$ denotes a column matrix in which $i_r = I_r$. $\{v\}$ denotes a column matrix in which $v_r = V_r - V_{2n-r+1} \cdot e^{i\beta l}$. Note that dipole r is connected to dipole $2n-r+1$. I_r and V_r are the current and free-space open-circuit voltage, respectively, in dipole r ($r = 1, 2, \dots, 2n$). Here, we have assumed that all transmission lines have the same length (l). For $n = 2$, Equations (15) corresponds to Equation (11) - (14) when $l_1 = l_2 = l$.

Let us now return to the reflector consisting of four parallel half-wave dipoles (see Fig. 4). An xy -coordinate system is introduced with its origin O and the direction of the x -axis conveniently selected. The four dipoles are placed with their centers in the xy -plane and their axes normal to the xy -plane. The reradiation pattern will be determined in the xy -plane when a plane wave is incident from an arbitrary direction in the xy -plane.

The induced voltages in the four dipoles are

$$V_1 = V_t e^{ip_{v1}} \quad (19)$$

$$V_2 = V_t e^{ip_{v2}} \quad (20)$$

$$V_3 = V_t e^{ip_{v3}} \quad (21)$$

$$V_4 = V_t e^{ip_{v4}} , \quad (22)$$

where V_t is the induced open-circuit voltage in a reference antenna placed at 0. p_{v1} , p_{v2} , p_{v3} , and p_{v4} are the phase differences between the induced voltages in dipoles 1, 2, 3, and 4, respectively, and the induced voltage in the reference antenna.

The currents I_1 , I_2 , I_3 , and I_4 may now be found from Equations (11) - (14). The electric field intensity at a distance r from 0 in a direction making an angle ϕ_u with respect to the x-axis is (3)

$$E = \frac{-i\xi}{2\pi} \frac{e^{ikr}}{r} (I_1 e^{ip_{i1}} + I_2 e^{ip_{i2}} + I_3 e^{ip_{i3}} + I_4 e^{ip_{i4}}) , \quad (23)$$

where ξ is the characteristic impedance of free space and k the free space propagation constant. p_{i1} , p_{i2} , p_{i3} , and p_{i4} are the phase differences (in the direction considered) between the dipoles 1, 2, 3, and 4, respectively, and the origin due to the differences in path lengths to the field point.

2.3. Condition for Van Atta effect

In this section the condition for Van Atta effect will be stated. When the coupling between the dipoles is neglected, the circumstances in which the condition is fulfilled will be derived.

From Equation (23) for the electric field intensity it is seen that the reradiation pattern will have a maximum in the direction for which the four terms in the factor

$$I_1 e^{ip_{i1}} + I_2 e^{ip_{i2}} + I_3 e^{ip_{i3}} + I_4 e^{ip_{i4}} \quad (24)$$

are in phase, that is when

$$\phi_{i1} + p_{i1} = \phi_{i2} + p_{i2} = \phi_{i3} + p_{i3} = \phi_{i4} + p_{i4} , \quad (25)$$

where ϕ_{i1} , ϕ_{i2} , ϕ_{i3} , and ϕ_{i4} are the arguments of I_1 , I_2 , I_3 , and I_4 , respectively. Usually there will not be any direction for which the conditions expressed in Equations (25) are fulfilled.

If we wish the reflector to have a Van Atta effect, Equations (25) must be satisfied back in the direction of incidence, for all angles of incidence.

In what follows we suppose, if other is not stated, that the dipoles in each pair of dipoles are placed symmetrically about the center O. Then we have

$$P_{i1} = P_{v1} = -P_{i4} = -P_{v4} \quad (26)$$

$$P_{i2} = P_{v2} = -P_{i3} = -P_{v3} \quad (27)$$

when we consider the reradiation back in the direction of arrival of an incident wave.

From this it appears that, if

$$\phi_{i1} = P_{v4} + u \quad (28)$$

$$\phi_{i2} = P_{v3} + u \quad (29)$$

$$\phi_{i3} = P_{v2} + u \quad (30)$$

$$\phi_{i4} = P_{v1} + u \quad (31)$$

then Equations (25) are satisfied. u is an arbitrary constant. If Equations

(28) - (31) are satisfied, then the ratios $\frac{I_1}{V_4}$, $\frac{I_2}{V_3}$, $\frac{I_3}{V_2}$, and $\frac{I_4}{V_1}$ have the same argument.

Let us state the result obtained above in the following manner: A sufficient condition for maximum reradiation back in the direction of arrival of an

incident wave is that $\frac{I_1}{V_4}$, $\frac{I_2}{V_3}$, $\frac{I_3}{V_2}$, and $\frac{I_4}{V_1}$ have the same argument. If this condition is satisfied for an arbitrary direction of incidence, the reflector has a Van Atta effect. (Note that if coupling and scattering were not present, then the reflector would have a Van Atta effect).

It is not a simple matter to see in which circumstances the condition for Van Atta effect is satisfied. However, a simple condition may be derived if we neglect the coupling and assume that the transmission lines have the same length

2. Then, if we introduce $Z_B = Z_{B1} = Z_{B2}$ and $Z_C = Z_{C1} = Z_{C2}$, Equations (1) - (4) reduce to

$$(Z_{An} + Z_B + Z_C)I_1 + Z_C I_4 = V_1 \quad (32)$$

$$(Z_{An} + Z_B + Z_C)I_2 + Z_C I_3 = V_2 \quad (33)$$

$$(Z_{An} + Z_B + Z_C)I_3 + Z_C I_2 = V_3 \quad (34)$$

$$(Z_{An} + Z_B + Z_C)I_4 + Z_C I_1 = V_4 \quad (35)$$

It is seen that if

$$Z_{An} + Z_B + Z_C = 0 \quad (36)$$

we have the Van Atta effect. Using Equations (9) and (10), the Equation (36) may be written

$$R_{An} + i(X_{An} + Z_0 \cot \beta l) = 0 \quad (37)$$

where $Z_{An} = R_{An} + iX_{An}$ is introduced. Since $R_{An} > 0$, it appears that Equation (37) cannot be satisfied. R_{An} may be decreased (reradiation increased) by realization of negative real loads (4); however, the reflector then is active and, in the present work, the investigation of active Van Atta reflectors is not included.

However, from Equations (32) - (35) and Equation (37) we may expect the following:

1. For an arbitrary value of Z_C , the reflector will have a smaller deviation from Van Atta effect if X_{An} is chosen so that $X_{An} + Z_0 \cot \beta l$ is zero, than if this is not the case.

2. The deviation from Van Atta effect decreases if $|Z_C|$ can be increased relative to $|Z_{An} + Z_B + Z_C|$. This may be attained by increasing $|Z_C|$ and, at the same time, by choosing X_{An} so that $X_{An} + Z_0 \cot \beta l = 0$.

The second method for decreasing the deviation from Van Atta effect has the drawback that the reradiation decreases as $|Z_C|$ increases and vanishes when $|Z_C| \rightarrow \infty$. This may be easily derived from Equations (32) - (35). Therefore, when we try to decrease the deviation from Van Atta effect by increasing $|Z_C|$, we have to take into account the restriction that the reradiation should not

decrease below some level dependent upon the application of the reflector.

It should be mentioned that when $|X_{An} + Z_0 \cot \beta l|$ is decreased by changing Z_0 or l , $|Z_C|$ might also decrease in such a way that a decrease in the deviation from Van Atta effect not is obtained.

It is to be noted that the condition (37) is obtained from the conditions (25) by neglecting coupling. Therefore, because coupling is always present the condition is only approximate. When coupling is taken into account, an exact condition simpler than the conditions (25) does not seem to exist.

It is not expected that it will be possible to find a reflector so that the conditions (25) are satisfied back in the direction of arrival of an incident wave, for all angles of incidence. However, it is expected that it should be possible to choose the lengths of the transmission lines, the location of the dipoles, the characteristic impedance of the transmission lines, and an impedance transforming network changing X_{An} so that the reflector has a "Van Atta effect to some extent". In the next section a measure of the deviation from Van Atta effect will be established.

2.4. Evaluation of the deviation from Van Atta effect

The object of this section is, for an arbitrary Van Atta reflector, to present a method for computing a figure which may be used as a measure of the deviation from Van Atta effect. The method is given for a reflector consisting of four antenna elements, but is easily extended to reflectors consisting of an arbitrary number of elements.

In order to find a measure of the deviation from Van Atta effect, we will consider a wave incident from a particular direction. The derivation is made by using a "complex current plane" (see Fig. 5). Let the points 1, 2, 3, and 4 on the unit circle be the points with arguments $\phi_{i1} + p_{i1}$, $\phi_{i2} + p_{i2}$, $\phi_{i3} + p_{i3}$, and $\phi_{i4} + p_{i4}$, respectively.

The magnitude of the currents is taken into account by placing $|I_1|$, $|I_2|$, $|I_3|$, and $|I_4|$ as "weights" at the points 1, 2, 3, and 4, respectively. The coordinates of the "center of inertia", C_r and C_i , are

$$C_r = \frac{|I_1| \cos(\phi_{i1} + p_{i1}) + |I_2| \cos(\phi_{i2} + p_{i2}) + |I_3| \cos(\phi_{i3} + p_{i3}) + |I_4| \cos(\phi_{i4} + p_{i4})}{|I_1| + |I_2| + |I_3| + |I_4|} \quad (38)$$

$$C_i = \frac{|I_1| \sin(\phi_{i1} + p_{i1}) + |I_2| \sin(\phi_{i2} + p_{i2}) + |I_3| \sin(\phi_{i3} + p_{i3}) + |I_4| \sin(\phi_{i4} + p_{i4})}{|I_1| + |I_2| + |I_3| + |I_4|} \quad (39)$$

As a measure of how much the fields from the currents are out of phase in the direction considered the "normalized momentum with respect to the center of inertia" $d_{\phi i}$ may be used. $d_{\phi i}$ is given by the formula:

$$d_{\phi i} = (|I_1| \sqrt{(C_r - \cos(\phi_{i1} + p_{i1}))^2 + (C_i - \sin(\phi_{i1} + p_{i1}))^2} + |I_2| \sqrt{(C_r - \cos(\phi_{i2} + p_{i2}))^2 + (C_i - \sin(\phi_{i2} + p_{i2}))^2} + |I_3| \sqrt{(C_r - \cos(\phi_{i3} + p_{i3}))^2 + (C_i - \sin(\phi_{i3} + p_{i3}))^2} + |I_4| \sqrt{(C_r - \cos(\phi_{i4} + p_{i4}))^2 + (C_i - \sin(\phi_{i4} + p_{i4}))^2}) / (|I_1| + |I_2| + |I_3| + |I_4|). \quad (40)$$

It is readily seen that $d_{\phi i}$ possesses the following properties:

1. $d_{\phi i}$ is zero if the points 1, 2, 3, and 4 are mapped into the same point. This is in accordance with the fact that, in this case, conditions (25) are satisfied and we have maximum reradiation in the direction considered.

2. $d_{\phi i}$ is not changed if all the currents are multiplied by the same factor. This should be the case since, in this circumstance, the shape of the reradiation pattern does not change.

In the above derivation we have considered a particular direction. In order to find a measure of the deviation from Van Atta effect, we have to take into account all directions of incidence. Strictly, this should involve an integration over all angles of incidence. However, it is assumed to be sufficient to consider the directions $\phi_i = 0^\circ, 10^\circ, \dots, 90^\circ$. Therefore, we use as a measure of the deviation from Van Atta effect the sum of the $d_{\phi i}$'s for the directions mentioned. This sum will be called d .

In Fig. 6 are given some illustrations of the information which the magnitude of $d_{\phi i}$ provides. The reradiation is calculated in an interval about the direction of incidence, since $d_{\phi i}$ only tells us about the reradiation near this direction. The examples are taken rather arbitrarily from different reradiation patterns. Owing to the fact that $d_{\phi i}$ does not indicate the magnitude of reradiation, a normalization is chosen so that the maxima of the beams nearest the Van Atta direction all have the same value. For convenience, the Van Atta direction is turned so that it is the same for all the patterns shown.

v is the angle between the Van Atta direction and the direction of the maximum nearest the Van Atta direction.

b indicates the reradiation in the Van Atta direction relative to the reradiation which would be obtained if all currents were in phase in this direction.

From the illustrations we observe that:

1. As $d_{\phi i}$ increases, b usually decreases as may be expected. An exception occurs in the case where $d_{\phi i}$ increases from 0.59 to 0.60 and b increases also. However, we notice that v is smaller for $d_{\phi i} = 0.59$ than for $d_{\phi i} = 0.60$.
2. When $d_{\phi i}$ is zero the currents are in phase in the Van Atta direction, $b = 100\%$, and $v = 0^\circ$.
3. As $d_{\phi i}$ increases, v tends to increase. The increment in v is larger for broad beams than for narrow beams. This may be expected since a large value of v will cause a larger decrement in the reradiation in the Van Atta direction in the case of narrow beams than for broad beams.

We have seen above that the value of $d_{\phi i}$ does not indicate:

1. The magnitude of reradiation back in the direction of incidence.
2. The magnitude of b . An exception occurs when $d_{\phi i} = 0$; then $b = 100\%$, and conversely.
3. The magnitude of the maximum of reradiation nearest the direction of incidence relative to the magnitude of the reradiation back in the direction of incidence.
4. The magnitude of v . An exception occurs when $d_{\phi i} = 0$; then $v = 0^\circ$. The converse may not be true.
5. The beam width of the beam in the Van Atta direction.
6. Anything about the reradiation pattern in directions which are not near the Van Atta direction.
7. That the back-scattering may be small even though $d_{\phi i}$ is very small. See Section 2.3.

Because knowledge of $d_{\phi i}$ tells us nothing about so many characteristics of the reradiated field, we may now ask ourselves whether it is worthwhile to try to find a Van Atta reflector with a small value of d or whether it would be better to find a reflector with large back-scattering for all angles of incidence. In fact we will do both.

First, it is decided to use the method developed for evaluation of the deviation from Van Atta effect, because the idea of the Van Atta reflector is that the fields from the antennas should be in phase back in the direction of incidence, and it would be interesting to see if it is possible to find a reflector with Van Atta effect.

Next, when we wish to use the reflector we may be more interested in large back-scattering. Therefore, we will also find the parameters of the reflector for which the back-scattering, as a function of the angle of incidence, has its largest minimum value.

The desirability of using $d_{\phi i}$ rather than b or v for evaluating the deviation from Van Atta effect may be discussed:

a. As mentioned above v may be zero even though the currents are not in phase. Because of this the reradiation may be small when v is zero. This often happens at endfire, since the reradiation pattern is symmetrical about the plane of the reflector. Therefore, we do not use v .

b. Considering Fig. 6 above, we observed that as $d_{\phi i}$ increases, b usually decreases. Hence, since $d_{\phi i} = 0$ is equivalent to $b = 100\%$, it may be expected that if we were to use b as a measure of the deviation from Van Atta effect, we would obtain results which would be as "good" as those found when we use d . However, because of exceptions similar to the above-mentioned in which b increases as $d_{\phi i}$ increases from 0.59 to 0.60 but v is smaller at 0.59 than at 0.60, we expect that the use of d as a measure of the deviation from Van Atta effect possesses a small advantage over the use of b .

It should be noted that a knowledge of d tells us about the shape of the reradiation pattern near the Van Atta direction but nothing about the magnitude of the back-scattering. Therefore, we can not use $d = 0$ as a criterion for large back-reradiation. However, the purpose of a Van Atta reflector is to have a large back-reradiation. Therefore, besides considering d , we will also consider the dependence of the back-scattering upon the parameters of the reflector.

2.5. Conditions for maximum back-scattering

In Section 2.3 it has been found that, for a given value of Z_C , the deviation from Van Atta effect is smaller if $X_{An} + Z_0 \cot \beta l = 0$, than if this is not the case. Furthermore, the deviation from Van Atta effect may be decreased by increasing $|Z_C|$ relative to $|Z_{An} + Z_B + Z_C|$, but at the same time the reradiation decreases.

The object of this section is to consider the manner in which the reradiation back in the direction of incidence depends on the value of $|Z_C|$ when $X_{An} + Z_0 \cot \beta l = 0$ and coupling is neglected.

To do this, and for the illustrations in subsequent sections, it is convenient to introduce the three quantities \max_a , g_a , and \min_a . These are, respectively, the maximum, the average, and the minimum values of the back-scattered field intensity as a function of the angle of incidence. The three quantities are assumed to be the most characteristic when we are interested in the back-scattered energy from endfire to broadside. From the maximum value we can see how large the back-scattered energy may be for a particular direction of incidence. When we know the average value, it may be expected that in some inter-

vals the reflector has back-scattered energy larger than g_a . If we are interested in a back-scattered energy above a particular level for all angles of incidence, we have to consider the magnitude of \min_a .

When $X_{An} + Z_0 \cot \beta l = 0$, the Equations (32) - (35) reduce to

$$R_{An} I_1 + Z_C I_4 = V_1 \quad (41)$$

$$R_{An} I_2 + Z_C I_3 = V_2 \quad (42)$$

$$R_{An} I_3 + Z_C I_2 = V_3 \quad (43)$$

$$R_{An} I_4 + Z_C I_1 = V_4 \quad (44)$$

By using Equations (19) - (23), (26), (27), and (41) - (44), the electric field intensity at a distance r from 0 in the Van Atta direction is found to be

$$E = -\frac{i\epsilon}{2\pi} \frac{e^{ikr}}{r} V_t \frac{2R_{An}(\cos 2p_{v1} + \cos 2p_{v2}) - 4Z_C}{R_{An}^2 - Z_C^2} \quad (45)$$

As a measure of the back-scattered field intensity we use

$$g_b = \frac{2\sqrt{r_{An}^2 (\cos 2p_{v1} + \cos 2p_{v2})^2 + 4Z_C^2}}{r_{An}^2 + Z_C^2} \quad (46)$$

where r_{An} and z_C are R_{An} and $|Z_C|$, respectively, normalized to 100 ohms. In Scientific Report no. 2, $g_b/2$ was used. It is seen that the value of g_b depends on the magnitude of $(\cos 2p_{v1} + \cos 2p_{v2})^2$, i.e., on the geometrical configuration of the dipoles. We have to consider two cases.

First, when the dipoles are placed so that the expression $(\cos 2p_{v1} + \cos 2p_{v2})^2$ can take the values 4 and 0, then g_b will have, respectively, a maximum value of

$$\frac{4}{\sqrt{r_{An}^2 + Z_C^2}} \quad \text{and a minimum value of} \quad \frac{4Z_C}{r_{An}^2 + Z_C^2} \quad . \quad \text{In these cases it is easily}$$

shown that the largest maximum and minimum values are $\frac{4}{r_{An}}$ and $\frac{2}{r_{An}}$, respectively. These two values are obtained when $|Z_C|$ equals zero and R_{An} , respectively. If we are interested in as large a value of \min_a as possible for all angles of incidence, then we have to choose Z_0 and l so that $|Z_C| = R_{An}$. Then the ratio of \max_a to \min_a is $\sqrt{2}$.

Next, the dipoles are placed so that the expression $(\cos 2p_{v1} + \cos 2p_{v2})^2$ can take at most one of the values 0 and 4. This may happen when the dipoles are close together. For example, in the not-realizable case for which all dipoles are placed in 0; here, g_b will always take its maximum value.

For a linear reflector with the elements equispaced, let us derive the spacings for which we have the second case mentioned above. If the spacing is a and the direction of incidence makes an angle ϕ_i with respect to the line on which the dipoles are placed, then

$$(\cos 2p_{v1} + \cos 2p_{v2})^2 = (\cos(3kac\phi_i) + \cos(kac\phi_i))^2, \quad (47)$$

where $k = 2\pi/\lambda$ and λ is the wavelength. Here $(\cos 2p_{v1} + \cos 2p_{v2})^2$ takes the value 4 for $\phi_i = 90^\circ$, independently of a . When the spacing is less than $\lambda/8$, the value zero is never obtained. Hence, when $0 < a < \frac{\lambda}{8}$, we have an example of a reflector which falls into the second category described above. From this it appears that if we desire \min_a to be large for a linear reflector, we must bring the dipoles as close together as possible. But then difficulties might be encountered in fabrication of small dipoles and due to coupling which has a large influence when the spacing, as here, is less than $\lambda/8$. This will be illustrated numerically in the next sections.

In this section we have found that if we wish to have no difficulties due to the dimensions of dipoles and due to coupling, and if we desire \min_a to be as large as possible, then we should choose $|Z_C|$ equal to R_{An} when $X_{An} + Z_C \cot \beta l = 0$.

It should be mentioned that \min_a could not be increased over the maximum value found above if we permitted $X_{An} + Z_C \cot \beta l \neq 0$.

2.6. The specular reflection

In this section we shall consider the specular reflection of a linear reflector. The coupling will be neglected. Specular effect is defined in the same way as Van Atta effect.

For the linear reflector considered in Section 2.5, it is found in a way similar to the above, that

$$g_s = \frac{2\sqrt{4r_{An}^2 + z_C^2} \{ \cos(3kac\phi_i) + \cos(kac\phi_i) \}^2}{r_{An}^2 + z_C^2} \quad (48)$$

may be used as a measure of the field intensity in the specular direction. The expression for g_s shows that its maximum value is equal to the maximum of g_b , but the minimum value usually differs from the minimum value of g_b . If we de-

sire a reflector with a small value of g_s , then we must increase $|Z_C|$; but this implies that the back-scattered energy decreases, as we have seen before.

In Scientific Report no. 2, R_{An} , X_{An} , and Z_0 have such values that the re-radiation pattern is symmetrical about the normal to the reflector when coupling is neglected. In general this is the case when

$$(R_{An})^2 + (X_{An} + Z_0 \cot \beta l)^2 = \frac{Z_0^2}{\sin^2 \beta l}. \quad (49)$$

If the condition $X_{An} + Z_0 \cot \beta l = 0$ is satisfied and $|Z_C| = R_{An}$, condition (49) is satisfied. Therefore, when we wish a small value of d , and \min_a to be maximum, the deviation from specular effect is equal to the deviation from Van Atta effect.

3. THE DEPENDENCE OF THE RERADIATION PATTERN ON THE PARAMETERS OF THE REFLECTOR

In this chapter some numerical illustrations of the foregoing theoretical results will be given. We consider the linear reflector consisting of four equispaced parallel half-wave dipoles. It will be shown in Section 3.1 that the method for evaluating the deviation from Van Atta effect provides results equivalent to those obtained by direct comparison between the reradiation patterns. In Section 3.2 we shall verify the conjecture that, for given Z_C , d is smaller if $X_{An} + Z_0 \cot \beta l$ is zero than if this is not the case. Furthermore, consideration will be given to that which occurs when we try to decrease d by increasing $|Z_C|$ relative to $|Z_{An} + Z_B + Z_C|$. In Section 3.3 the optimum values of Z_0 , l , and X_{An} will be found and in Section 3.4 the optimum value of a will be determined. The optimum values are defined as the values for which \min_a , as a function of Z_0 , l , X_{An} , and a , has its flattest maximum and d is as small as possible. In Section 3.5 the effects of some particular variation of the parameters of specific reflectors will be described. Finally, in Section 3.6, we will see what happens when the transmission lines are of unequal lengths, the dipoles are not equispaced, and the reflector is not linear.

Throughout the investigation the influence of coupling will be estimated. The deviation between the curves for which coupling is taken into account and those for which coupling is neglected are due to the fact that coupling will influence the magnitude and the phase of the currents in the dipoles. Since this influence depends on the spacing between the elements in a complicated manner, its magnitude will only be estimated from the numerical results in each case. No attempt will be made to give a detailed explanation for the deviations between the two sets of curves. Such an explanation could be given by considering the magnitudes of the mutual impedances.

3.1. Illustration of the method for evaluating the deviation from Van Atta effect

In Fig. 7 the measure of the deviation (d) from Van Atta effect is shown as a function of the length (l) of the transmission lines in the case for which $a = 0.50\lambda$, $X_{An} = 0$ ohms, and $Z_0 = 73$ ohms. From the figure it is found, when coupling is neglected, that d takes its minimum value for $l = 0.25\lambda + j0.50\lambda$ and its largest values for $l = 0.50\lambda + j0.50\lambda$. The fact that the minimum value occurs for $l = 0.25\lambda + j0.50\lambda$ is in accordance with the condition (37).

When coupling is taken into account, we see that the smallest value of d occurs for $l = 0.28\lambda + p\lambda$, and that coupling disturbs the symmetry with respect to $l = 0.50\lambda$. Furthermore, for $l = 0.28\lambda + p\lambda$, coupling decreases d so that it becomes smaller than the minimum value of d obtained when coupling is neglected. Usually, however, coupling increases the deviation from Van Atta effect.

In order to compare the reradiation patterns directly, these have been calculated for $\phi_i = 0^\circ, 10^\circ, \dots, 90^\circ$ for several lengths of the transmission lines when a , X_{An} , and Z_0 take the values given above. For each length we have evaluated the angle between the Van Atta direction and the direction of the maximum nearest the Van Atta direction for $\phi_i = 0^\circ, 10^\circ, \dots, 90^\circ$. The sum of these ten angles, as a function of the lengths, takes its smallest value for $l = 0.25\lambda + p0.50\lambda$ when coupling is neglected and for $l = 0.28\lambda + p\lambda$ when coupling is taken into account (compare with Scientific Report no. 2, p.14).

The agreement between the above mentioned results indicates that the method for evaluating the deviation from Van Atta effect may be used when many reflectors are to be compared. The evaluation of d is much easier than the determination of the angle between the Van Atta direction and the direction of the maximum of reradiation nearest the Van Atta direction. The evaluation of d requires only a knowledge of the currents in the dipoles whereas the evaluation of the above-mentioned angle requires in addition a knowledge of the shape of the reradiation pattern. Furthermore, as discussed in Section 2.4, the above-mentioned angle may be zero even when the currents are not in phase.

3.2. The condition for decreasing the deviation from Van Atta effect

In Section 2.3 it has been seen, when coupling between the dipoles is neglected, that we may expect to have, for constant Z_C , a smaller d if

$$X_{An} + Z_0 \cot \beta l = 0, \quad (50)$$

than if this is not the case. Furthermore, it was found that we can decrease d by increasing $|Z_C|$ relative to $|Z_{An} + Z_B + Z_C|$. In fact, d can be decreased as much as we desire but then, unfortunately, the reradiation decreases. In this section, these results will be verified by considering some numerical computations.

3.2.a. The condition $X_{An} + Z_0 \cot \beta l = 0$

In Figs. 8 and 9, d is plotted as a function of l when Z_0 equals 73 ohms and 50 ohms, respectively. For each value of Z_0 two curves are shown, one for which X_{An} is chosen so that $X_{An} + Z_0 \cot \beta l = 0$ for each value of l and one for

which $X_{An} = 0$ ohms. We see that the two curves meet for $\ell = 0.25\lambda$ since then the condition $X_{An} + Z_0 \cot \beta \ell = 0$ requires $X_{An} = 0$ ohms. As expected we notice that the deviation from Van Atta effect is smaller if $X_{An} + Z_0 \cot \beta \ell = 0$, than if this is not the case. When $Z_0 = 73$ ohms and $X_{An} = 0$ ohms, the smallest value of d , as a function of ℓ , occurs for $\ell = 0.25\lambda + p0.50\lambda$ since then $X_{An} + Z_0 \cot \beta \ell = 0$. This is not the case when $Z_0 = 50$ ohms and $X_{An} = 0$ ohms. Here d , as a function of ℓ , has a maximum when $X_{An} + Z_0 \cot \beta \ell = 0$. This may be explained in the following way: as ℓ increases or decreases from $0.25\lambda + p0.50\lambda$ then $|Z_C|$ increases. This decreases d to a greater extent than d is increased by the increase in $|X_{An} + Z_0 \cot \beta \ell|$.

When Z_0 is about 73 ohms or larger, then d , as a function of ℓ , will have a minimum when $X_{An} + Z_0 \cot \beta \ell = 0$. This is illustrated in Figs. 10 and 11, where d is shown as a function of the length of the transmission lines when X_{An} is equal to -73, 0, and 73 ohms. When coupling is neglected, it is seen that, in accordance with condition (50), d has minima for $\ell = 1/8, 2/8, 3/8\lambda$ in the interval 0λ to $1/2\lambda$ and for $\ell = 5/8, 6/8, 7/8\lambda$ in the interval $1/2\lambda$ to 1λ when $X_{An} = -73, 0$, and 73 ohms, respectively. Coupling between the dipoles produces only some slight changes in the positions of the minima.

3.2.b. The decrement of d

We shall now see what happens when we try to decrease d by increasing Z_C if, at the same time, the condition (50) is satisfied. The increment of Z_C may be obtained by letting (a) Z_0 tend to infinity, or (b) ℓ tend to a multiple of half a wavelength, or (c) Z_0 and ℓ tend, in an arbitrary manner, to infinity and a multiple of half a wavelength, respectively. This method which is a combination of (a) and (b) will not be examined.

a. In Fig. 12, d is shown as a function of Z_0 when $a = 0.50\lambda$, $X_{An} = 0$ ohms, and $\ell = 0.25\lambda$. Since $X_{An} + Z_0 \cot \beta \ell = 0$, the deviation from Van Atta effect takes its largest value for $Z_0 = 0$ ohms (the dipoles short-circuited) and decreases when Z_0 is increased; but, as expected, the reradiation decreases (see Figs. 13.a, 13.b, 13.c). In these figures and in what follows \max_a , g_a , and \min_a denote, respectively, the maximum, the average, and the minimum values of the back-scattered field intensities for the angles of incidence $0^\circ, 10^\circ, \dots, 90^\circ$. Since the curve for \max_a is above that for g_a and the curve for \min_a is below, it is not indicated on the figures which curve refers to which quantity.

Above we noticed that as Z_0 tends to infinity (dipoles open-circuited) g_a decreases. On the other hand g_a is also decreased when Z_0 tends to zero (dipoles

short-circuited). Since $|Z_C| = Z_0$ for the length chosen, \min_a has its maximum for $Z_0 = 73$ ohms. Furthermore, \max_a takes its largest value for $Z_0 = 0$ ohms (see Section 2.5). Since g_a is a measure of the average back-scattering, g_a has its maximum for a value of Z_0 between 0 ohms and 73 ohms.

We see that coupling increases d and decreases g_a . The maximum of \min_a occurs at about $Z_0 = 60$ ohms when coupling is taken into account. Furthermore, we see that due to coupling \min_a is not zero when the transmission lines are short-circuited.

b. In Figs. 14, 15.a, 15.b, and 15.c are shown what happens if we try to decrease the deviation from Van Atta effect by letting ℓ tend to a multiple of half a wavelength. For all values of ℓ we have chosen X_{An} so that $X_{An} + Z_0 \cot \beta \ell = 0$. In Fig. 14, d is plotted as a function of ℓ when $X_{An} = -Z_0 \cot \beta \ell$, $Z_0 = 73$ ohms, and the equispacing $a = 0.50\lambda$. Owing to the fact that $|Z_C|$ is smallest for $\ell = 0.25\lambda + p0.50\lambda$, d is largest for $\ell = 0.25\lambda + p0.50\lambda$. As ℓ tends to a multiple of half a wavelength $|Z_C|$ increases and, therefore, d decreases. However, at the same time the reradiation decreases and is zero when ℓ equals a multiple of half a wavelength. This is in accordance with the remarks in Section 2.3 and is illustrated in Figs. 15.a, 15.b, and 15.c.

From Fig. 14 we see that coupling increases the deviation from Van Atta effect for all lengths of the lines. Figs. 15.a, 15.b, and 15.c show that in the interval 0λ to 0.50λ coupling decreases g_a and in the interval 0.50λ to 1λ , g_a is enhanced. Note that in the interval from 0.50λ to 1λ , coupling increases \max_a but decreases \min_a . See Section 3.5 for what happens when ℓ is equal to a multiple of half a wavelength and X_{An} not chosen so that $X_{An} + Z_0 \cot \beta \ell = 0$.

3.3. The optimum values of Z_0 , ℓ , and X_{An} when coupling is neglected

In this section we will find the optimum values of Z_0 , ℓ , and X_{An} when coupling is neglected. According to the definition of optimum values of parameters we must find the values of Z_0 , ℓ , and X_{An} for which \min_a has its flattest maximum, and d is as small as possible. In Section 2.5 it has been found, when $a > \lambda/8$, that \min_a is largest when $|Z_C| = R_{An} = 73$ ohms and X_{An} chosen so that $X_{An} + Z_0 \cot \beta \ell = 0$. However, there are infinitely many sets of values of Z_0 , ℓ , and X_{An} which satisfy the conditions $|Z_C| = R_{An}$ and $X_{An} + Z_0 \cot \beta \ell = 0$. For all such sets of values it follows, by considering Equations (32) - (35), that d is the same. Due to these facts, we shall try to find some optimum values so that some change in ℓ or Z_0 only causes the smallest change in \min_a . The determination will be made graphically when $a = 0.50\lambda$, but will be valid for all $a > \lambda/8$.

The values obtained will be called the optimum values. The graphical determination is made since this is more illustrative, than a mathematical determination obtained by considering Equation (45).

Since $Z_C = iZ_0/\sin\beta l$, we have to explore the variations in d and \min_a as functions of l and Z_0 when X_{An} satisfies $X_{An} + Z_0 \cot\beta l = 0$ for all values of Z_0 and l . Figs. 16 and 17 show the behaviour for Z_0 equal to 10, 30, 73, and 110 ohms. When $Z_0 > 73$ ohms, the maximum value of \min_a is less than that obtainable for smaller values of Z_0 since $|Z_C| > R_{An}$ for all values of l . When $Z_0 < 73$ ohms, \min_a has its maximum value when l is chosen so that $|Z_C| = R_{An}$. The least critical choice is $l = 0.25\lambda + p0.50\lambda$ when $Z_0 = 73$ ohms, since then the maximum of \min_a , as a function of l , is more flat than when $Z_0 < 73$ ohms. Hence, we take 73 ohms and $0.25\lambda + p0.50\lambda$ as the optimum values of the characteristic impedance and the length of the transmission lines, respectively. Then, from the condition $X_{An} + Z_0 \cot\beta l = 0$, it follows that 0 ohms is the optimum value of X_{An} .

In Fig. 16 we see that d increases as Z_0 decreases. This is in accordance with Section 2.3.

3.4. The optimum value of a when coupling is neglected

In Section 2.5 it has been found that the maximum value of \min_a is independent of the spacing when this is larger than $\lambda/8$. Below it will be seen why we are not interested in $a < \lambda/8$. Hence, we cannot optimize a with respect to \min_a . Instead we will now optimize a with respect to d when coupling is neglected and when Z_0 , l , and X_{An} have the optimum values found above. The optimization will be performed numerically and only spacings less than three wavelengths will be considered. Hence, we will find the smallest value of d for $\lambda/8 < a < 3\lambda$.

In Fig. 18, d is plotted as a function of a . We notice that d has its minima when a is about a multiple of a quarter-wavelength. An explanation of this has not been found. When a is less than one wavelength, the minima are usually more pronounced than when a is larger than one wavelength. The smallest values of d occur at a multiple of half a wavelength. The optimum value of a is at one and a half wavelength when coupling is neglected. Usually coupling increases the deviation from Van Atta effect.

The variation in d with spacing is also explored when Z_0 , l , and X_{An} do not have their optimum values. An example is shown in Fig. 19. It turns out that, usually, d still has minima when a is about a multiple of half a wavelength.

When the spacing between dipoles is increased, coupling decreases. This is illustrated in Figs. 18 and 20. a from which it appears that as the spacing increases, the deviation decreases between the curves for which coupling is neglected and those for which coupling is taken into account.

In accordance with Section 2.5, Fig. 20.b shows that \max_a is constant for all values of a . \min_a is constant for $a > \lambda/8$ but tends to \max_a when spacing tends to zero. When a is somewhat larger than a multiple of half a wavelength, g_a has a maximum.

From Fig. 20.b and 20.c we see that, when the spacing is less than one wavelength, coupling may change \max_a by as much as 50 % from the value it has when coupling is neglected.

Note that for $a < \lambda/8$, \min_a does not tend to $2\sqrt{2}/r_{An}$ when a tends to zero as is the case when coupling is neglected. Furthermore, the reradiation is rather small when a is less than half a wavelength. Because of this we are not interested in examining a reflector with $a < \lambda/8$.

3.5. Specific variations

In this section we shall consider the effect of some specific variations which could not be logically described in the foregoing sections, but which might be of some interest.

3.5.a. Dependence of X_{An}

In Figs. 21 and 22, d and g_a are plotted as functions of X_{An} for $Z_0 = 73$ ohms, $\ell = 0.25\lambda$, and $a = 0.50\lambda$. When coupling is neglected, d has a minimum for $X_{An} = 0$ ohms; this is in accordance with the fact that for the chosen reflector $X_{An} + Z_0 \cot \beta \ell = 0$ for $X_{An} = 0$ ohms. Coupling makes only a slight change in d but changes the maximum of g_a from occurring for $X_{An} = 50$ ohms to being for $X_{An} \approx -100$ ohms.

3.5.b. Independence of Z_0

The reradiation pattern does not change with variations in Z_0 when the lengths of the transmission lines are multiples of half a wavelength. This is proved in the following manner.

Let $\ell_1 = n_1 \frac{\lambda}{2}$ and $\ell_2 = n_2 \frac{\lambda}{2}$, where n_1 and n_2 are integers. From Equations (11) - (14) it can be shown that $I_4 = -I_1 e^{in_1 \pi}$ and $I_3 = -I_2 e^{in_2 \pi}$, and that I_1 and I_2 are determined by

$$(2Z_{An} - 2Z_{14} e^{in_1 \pi}) I_1 + (Z_{12} - Z_{24} e^{in_1 \pi} - Z_{13} e^{in_2 \pi} + Z_{34} e^{i(n_1+n_2)\pi}) I_2 = V_1 - V_4 e^{in_1 \pi} \quad (51)$$

$$(Z_{12} - Z_{24} e^{in_1 \pi} - Z_{13} e^{in_2 \pi} + Z_{34} e^{i(n_1+n_2)\pi}) I_1 + (2Z_{An} - 2Z_{43} e^{in_2 \pi}) I_2 = V_2 - V_3 e^{in_2 \pi} \quad (52)$$

From this it appears that the reradiation pattern is independent of the characteristic impedance Z_0 , when the lengths of the lines are multiples of $\lambda/2$. This may be readily understood by considering the equivalent X-circuit for the transmission lines. The result is illustrated in Figs. 23 and 24 where d and g_a are shown as functions of the length of the lines for various values of the characteristic impedance. It is seen that the curves meet at a common point when ℓ is a multiple of half a wavelength. Except in some small intervals, coupling increases the deviations from Van Atta effect and, except at one of the maxima when $Z_0 = 90$ and 73 ohms, the reradiation is decreased. Notice furthermore that coupling changes the relative magnitude of the maxima in the interval from 0.50λ to 1λ (see Fig. 24). These results are naturally only valid for the reflector with $a = 0.50\lambda$ and $X_{An} = 0$ ohms, but indicate how much coupling may influence the reradiation properties of a reflector. This is also shown in Figs. 25 and 26 from which it is seen that the maximum of \max_a is increased considerable by coupling.

3.5.c. Independence of X_{An}

When coupling is neglected, it is easily shown that the shape of the reradiation pattern does not change with X_{An} when the lengths of the transmission lines are multiples of half a wavelength (see Equations (51) and (52)). The reradiation will decrease as $|X_{An}| \rightarrow \infty$ and be maximum when $X_{An} = 0$ ohms. This is illustrated in Fig. 27 where g_a is shown as a function of X_{An} for $a = 0.50\lambda$, $\ell = 0.50\lambda$, and $Z_0 = 73$ ohms. d is found to be 5.64 . \min_a is zero since, for this specific choice of parameters, we have no reradiation at all for endfire.

3.6. Asymmetries in the reflector

In this section we shall consider, as examples, what happens when the transmission lines are of unequal length, the dipoles are not equispaced, and the reflector is not linear.

Let ℓ_1 and ℓ_2 be the lengths of the transmission lines and let n_{11} , n_{12} , n_{21} , n_{22} , n_{31} , n_{32} , n_{41} , and n_{42} be the coordinates of the dipoles (see Fig. 28).

As our starting point we will take the linear reflector with $a = 0.50\lambda$, $\ell = 0.25\lambda$, $X_{An} = 0$ ohms, and $Z_0 = 73$ ohms.

In Fig. 29, d is shown as a function of ℓ_2 when the other parameters are constant. The curve for which coupling is taken into account does not have a minimum for $\ell_2 = 0.25\lambda$, but it is seen that coupling only shifts its position by 0.05λ . As might be expected, d has its maximum when ℓ_2 deviates by $\lambda/2$ from ℓ_1 .

In Fig. 30, d is plotted as a function of n_{11} , i.e., the effect of non-equispacing is examined. In Fig. 31, d is shown as a function of n_{12} , i.e., the effect of non-linearity is investigated. In Figs. 30 and 31 it is seen that coupling shifts the position of the minima by about 0.02λ .

These examples show that in a Van Atta reflector in which coupling is present, the deviation from Van Atta effect may be decreased by permitting the transmission lines to be of unequal length, the dipoles to be non-equispaced, and the reflector to be non-linear. However, the decrement which may be obtained is small. Similarly, it is found that the increment in \min_a obtained by permitting the above-mentioned asymmetries is small.

4. NUMERICAL OPTIMIZATION

In the previous chapter we have found that it is possible to find values of the parameters so that the deviation from Van Atta effect d may be made as small as we desire. Unfortunately, the reradiation decreases in such a way that the smaller the value of d the smaller the reradiation will be.

Due to this fact we shall optimize the reradiation pattern of the Van Atta reflector by using two different criteria.

First, we shall find the parameters of the reflector for which \min_a is as large as possible, and d as small as possible.

Next, we shall find the parameters of the reflectors for which \min_a is above various prescribed levels, and d as small as possible.

In Chapter 3 we have used the first criterion to find the so-called optimum values of the parameters when coupling is neglected. Furthermore, the influence of coupling has been estimated for particular combinations of parameters. In this chapter coupling will be taken into account for a large number of combinations of parameter values and we are interested in exploring the possibility of finding values of l , a , X_{An} , and Z_0 for which coupling increases the value of \min_a over its maximum value $2/r_{An}$, obtained when coupling is neglected.

Similarly, by using the second criterion, we shall investigate whether or not coupling for some values of l , a , X_{An} , and Z_0 causes the reflector, for which \min_a is above a prescribed level, to have a deviation from Van Atta effect which is smaller than that obtained when coupling is neglected.

After the two criteria have been applied to the reflector with transmission lines of equal length and equispaced dipoles, a further optimization will be made in which we permit the transmission lines to be of unequal lengths, the dipoles to be non-equispaced, and the reflector to be non-linear.

4.1. The reflector with the largest value of \min_a , and d as small as possible

4.1.a. The method

In Chapter 3 it has been found when coupling is neglected, that the optimum values of l , a , X_{An} , and Z_0 are $0.25\lambda + p0.50\lambda$, 1.50λ , 0 ohms, and 73 ohms, respectively. Coupling may change this result. Since coupling has the greatest effect for $a < l\lambda$, we shall first try to obtain an idea of whether coupling in-

creases or decreases \min_a , for $a < 1\lambda$, by evaluating \min_a and d for all combinations of values of l , a , X_{An} , and Z_0 given in Table 1. Some characteristic sets of values are selected for l and a and the values chosen for X_{An} and Z_0 are near their optimum values ($X_{An} = -42$ ohms is the reactance of a half-wave dipole). In this way, \min_a and d are calculated for 1600 sets of parameters.

After considering the results of this calculation, we shall select the 10 sets with the largest values of \min_a . These 10 sets are then optimized as described in Section 4.1.c. Together with the 10 sets, the 2 sets with the optimum values given at the beginning of this section ($l = 0.25\lambda$ and 0.75λ) will be optimized. Hence, we get an idea of the influence of coupling when it is large ($a < 1\lambda$) and when it is small ($a = 1.5\lambda$). The maximum value of \min_a is not found since a continuous variation of the parameters is not made, but it is expected that the results will only deviate by a few per cent.

4.1.b. The results for $a < 1\lambda$

The results obtained by evaluating the above-mentioned 1600 reflectors are conveniently described by considering \min_a as a function of one parameter with the other held fixed.

It has been found that:

1. \min_a , as a function of l , usually has maxima near the lengths determined by $\cot \beta l = -X_{An}/Z_0$. A typical variation of \min_a as a function of the length is shown in Fig. 32.
In particular it is found:
 - a. \min_a is zero for $l = 1\lambda$.
 - b. \min_a is small for $l = 1/2\lambda$. In particular, \min_a is zero for $l = 1/2\lambda$ and $a = 1/2\lambda$ or 1λ .
2. \min_a , as a function of a , usually has its largest value for $a = 1\lambda$ and a small maximum near $a = 1/2\lambda$. For an example see Fig. 20.c. As mentioned above, exceptions to this behaviour occur for $l = 1/2\lambda$ and 1λ .
3. Any choice of X_{An} , from among the values of Table 1, gives essentially the same maximum value of \min_a by choosing appropriate values of l , a , and Z_0 .
4. \min_a , as a function of Z_0 , usually has a maximum for $Z_0 = 73$ ohms. However, there are some exceptions to this rule. One exception has already been shown in Fig. 13.c.

We may conclude that there have not been found any sets of values for the parameters of the reflector for which coupling causes variations in \min_a other than those already observed in Chapter 3. Furthermore, it has been found that

the 10 sets with the largest values of \min_a all have $a = 1\lambda$. From this we deduce that for all combinations of values of l , X_{An} , and Z_0 for which \min_a has a considerable value, coupling reduces \min_a as shown in Fig. 20.c.

4.1.c. The results of the optimization process

We will now consider the optimization of the 10 sets, together with the 2 optimum sets mentioned at the beginning of Section 4.1.a. The optimization is performed in the following way. First, l is increased or decreased in steps of 0.02λ until it is equal to a value for which \min_a , as a function of l , is as close as possible to a maximum. With the value of the length so obtained we, similarly, change a , X_{An} , and Z_0 in turn by steps of 0.02λ , 5 ohms, and 5 ohms, respectively. Then l , a , X_{An} , and Z_0 are changed anew in the same way, and so on, until no enhancement of \min_a occurs when the parameters are increased or decreased by the steps given above. The magnitude of the steps is chosen so that the optimization process does not take too much time and we may expect to obtain a value of \min_a which is close to its maximum.

In Table 2 the results are shown for the 12 reflectors which are optimized. In the rows denoted b and a we have the values of d , \min_a , and the parameters of the reflectors before and after the optimization, respectively. As explained above, a is equal to 1λ for the first 10 sets. We see that coupling causes l , X_{An} , and Z_0 to differ from the optimum values obtained when coupling is neglected, namely, $0.25\lambda + j0.50\lambda$, 0 ohms, and 73 ohms, respectively. The deviations are more pronounced for $a = 1\lambda$ than for $a = 1.5\lambda$ which is attributable to the fact that coupling is larger for 1λ than for 1.5λ . We see that the largest value of \min_a , which is obtained by the optimization, is 2.82 and occurs for $a = 1.54\lambda$ (compare with Fig. 20.c). Here we have an example in which coupling increases \min_a over the largest value of \min_a , 2.74, which is obtained if coupling is not present. However, the increment is negligible and some of it may be due to the fact that the back-scattered energy is only computed for discrete directions and, therefore, the exact value of \min_a is not obtained. For $a = 1\lambda$ the largest value of \min_a is 2.64, which is below the largest value 2.74 obtained when coupling is neglected.

The reradiation patterns for the two cases mentioned above are shown in Figs. 33 and 34. We see that for $a = 1.54\lambda$ there are more and sharper beams than for $a = 1\lambda$. In agreement with the curve in Fig. 18, the deviation from Van Atta effect is smaller for $a = 1.54\lambda$ than for $a = 1\lambda$ (see Table 3). A further discussion referring to these reradiation patterns will be given in Section 4.3.

4.2. Reflectors with \min_a over a prescribed level, and d as small as possible

4.2.a. The method

In this section we shall find the values of the parameters for which \min_a is larger than 1.5, 2.0, and 2.5, and d is as small as possible.

Again, when coupling is small ($a = 1.5\lambda$), we shall find how small a value of d there may be obtained by varying the parameters about the optimum values found when coupling is neglected.

When coupling is large ($a < 1\lambda$), from among the 1600 sets of parameters mentioned before, we select the 10 sets for which $Z_0 = 73$ ohms and d is smallest. It will be explained later why we have chosen the 10 sets with $Z_0 = 73$ ohms. These 10 sets together with the 2 optimum sets mentioned at the beginning of Section 4.1.a, are optimized as described below. Again, we do not obtain the minimum value of d when \min_a is above the prescribed level, but the evaluation provides an idea of whether or not it is possible to choose l , a , X_{An} , and Z_0 so that coupling causes the deviation from Van Atta effect to be less than that which would be obtained if coupling were not present.

4.2.b. The results for $a < 1\lambda$

From the analysis of the results of the 1600 reflectors we conclude the following for the deviation from Van Atta effect:

1. d , as a function of l , usually has minima near the lengths determined by $\cot \beta l = -X_{An}/Z_0$.
In particular, it is found that d usually has its largest value for $l = 1/2\lambda$ and 1λ .
2. d , as a function of a , has minima for $a = 1/2\lambda$ and 1λ . Often d has a minimum for $a = 0.75\lambda$ but it has rarely a minimum for $a = 0.25\lambda$. Hence, a typical variation is shown in Fig. 18.
3. Any choice of X_{An} , from among values in Table 1, gives essentially the same minimum value of d by choosing appropriate values of l , a , and Z_0 .
4. d , as a function of Z_0 , decreases as Z_0 increases.

As in Section 4.1.b we have not found any sets of values of the parameters of the reflector for which coupling causes variations in d other than those already observed in Chapter 3.

4.2.c. The results of the optimization process

We will now describe the optimization of the 12 sets mentioned in Section 4.2.a.

Above, we have seen that d is decreased as Z_0 increases. Therefore, it was decided to optimize reflectors for which $Z_0 = 73$ ohms and to finish the optimization process by increasing Z_0 . Hence, the optimization process has been made in two steps.

First, d is minimized by an optimization process in which ℓ , a , and X_{An} in turn change by steps of 0.02λ , 0.02λ , and 15 ohms, respectively, in a similar manner to that described in Section 4.1.c. Next, d is decreased by increasing Z_0 in steps of 20 ohms until \min_a is below 1.5. As Z_0 is increased we adjust X_{An} so that $X_{An} + Z_0 \cot \beta \ell = 0$, since we know from the previous chapter that the deviation from Van Atta effect is smaller when $X_{An} + Z_0 \cot \beta \ell = 0$, than if this is not the case.

The results are shown in Table 4 for the 12 sets. For every set the values of d , \min_a , the parameters of the reflector, and $X_{An} + Z_0 \cot \beta \ell$ are shown when:

- a. The optimization starts.
- b. The first step of optimization has been made.
- c. X_{An} has been adjusted after the first step of optimization so that $X_{An} + Z_0 \cot \beta \ell = 0$.
- d. The second step of optimization has been made.

The rows designated a, b, c, and d in the table correspond to the a, b, c, and d just mentioned. We see that a is either 0.50λ or 1λ for the 10 sets selected from among the 1600 examples. This is in accordance with the curve for d in Fig. 18 where coupling is taken into account. This curve has namely, as mentioned above, the smallest minima when a is equal to 0.50λ and 1λ . It is seen that, by the optimization process, a is not changed or only changes by 0.02λ .

As might be expected we see that after the first step of optimization $X_{An} + Z_0 \cot \beta \ell$ is decreased. Exceptions are the cases in which we start with $X_{An} + Z_0 \cot \beta \ell = 0$. This is due to coupling.

When X_{An} is adjusted after the first step of optimization so that $X_{An} + Z_0 \cot \beta \ell = 0$, we notice that d often is increased and not as might be expected decreased. However, the increment is small and is due to coupling. This has been confirmed by computation. Furthermore, computations have shown that it is not possible to obtain smaller values of d if X_{An} is not adjusted after the first step of optimization.

From the values of d and \min_a obtained by the optimization process, the parameters are selected for which \min_a is larger than 1.5, 2.0, 2.5, and d is

as small as possible. This is done for both the first 10 sets and the 2 optimum sets. The result is shown in Table 5. It appears that when we require a larger value of \min_a , then we find a larger value of d . It is seen that d is smaller for $a = 1.50\lambda$ than for $a = 1.00\lambda$ and 0.50λ (values of d corresponding to the same prescribed level of \min_a must be compared). These two results might be expected from the results in Chapter 3.

In Table 6 is shown the values of d which may occur if coupling is not present for $a = 1.50\lambda$. d is made small by increasing Z_0 . A comparison between Tables 5 and 6 shows that it has not been possible to find values of l , a , X_{An} , and Z_0 for which coupling decreases the deviation from Van Atta effect so that it is smaller than the deviation which may be obtained if coupling were not present.

Tables 5 and 6 show that when we prescribe $\min_a > 2.5$, then l , a , X_{An} , and Z_0 are close to their optimum values mentioned at the beginning of Section 4.1.a. If we prescribe \min_a to be above the levels 1.5 and 2.0, then d is decreased by increasing Z_0 .

In Fig. 35 is shown the reradiation patterns for the reflector with the smallest value of d when $\min_a > 2.5$, $a \leq 1\lambda$, and coupling is taken into account. This reflector has $a = 1\lambda$. In Fig. 36 the reradiation pattern is shown for the reflector with the smallest value of d when $\min_a > 2.5$, $a \leq 3\lambda$, and coupling neglected. This reflector has $a = 1.5\lambda$. As in the previous section we notice that there are more and sharper beams for $a = 1.5\lambda$ than for $a = 1\lambda$. A further discussion of the reradiation patterns follows in the next section.

4.3. The two types of reflectors

Table 3 shows the characteristic data of the reradiation patterns shown in Figs. 33 - 36. The parameters of the corresponding reflectors are also given. The plus signs indicate that coupling is taken into account and the minus signs imply that coupling is neglected. d_s is a measure of the deviation from specular effect in the same way as d is a measure of the deviation from Van Atta effect.

For the above reflectors we will discuss the influence of coupling and compare the deviation from specular effect with the deviation from Van Atta effect. Furthermore, we will discuss the differences between the reflectors obtained by the two methods of optimization.

It is observed that coupling decreases \min_a in all cases, except in the case shown in Fig. 34 and discussed above. A computation has shown that coupling decreases the average back-scattering, g_a , in all cases. From Fig. 35 we

see that coupling decreases the deviation from Van Atta effect whereas coupling in the other cases increases the deviation.

From the reradiation patterns it is seen that the reradiation in the specular direction is of the same magnitude as the reradiation in the Van Atta direction. This is in accordance with the results of Section 2.6. Table 3 shows that coupling increases the deviation from specular effect just as it usually increases the deviation from Van Atta effect.

For the two reflectors (in Table 3) optimized with respect to \min_a , the deviation from specular effect is smaller than the deviation from Van Atta effect, whereas for the two reflectors optimized with respect to d , the deviation from specular effect is larger than the deviation from Van Atta effect.

The largest difference between the deviation from Van Atta effect and the deviation from specular effect occurs in the case shown in Fig. 36. This may be seen from the reradiation patterns. For the ten patterns in Fig. 36, a direct computation has shown that the sum of the angles between the Van Atta direction and the direction of the maximum nearest the Van Atta direction is 14° . The corresponding sum of angles for the specular direction is 36° .

The difference in d between the two types of reflectors will be smaller the larger the value of \min_a we prescribe. For, if we prescribe \min_a to be large enough, namely 2.82 for the reflectors evaluated above, there would be no difference.

The optimization with respect to d demonstrates how small a deviation from Van Atta effect there may be obtained if we do not require \min_a to be as large as possible. In the example with $d = 1.66$, it has been found, by averaging over the 10 reradiation patterns shown in Fig. 36, that the reradiation in the Van Atta direction is 96 % of the reradiation which would be obtained if all the fields from the antennas were in phase in this direction.

For the reflector obtained by optimizing \min_a , it is found that the deviation from Van Atta effect is so small ($d = 2.53$) that only in one case does the maximum deviate by as much as 10° from the Van Atta direction and the deviation usually is less than a few degrees (see Fig. 34). Furthermore, the reradiation in the Van Atta direction is 92 % of the reradiation which would be obtained if all fields from the antennas were in phase. Thus, even the reflector obtained by optimizing \min_a may be said to have a small deviation from Van Atta effect.

4.4. Further optimization

From the reflectors described before we have chosen two reflectors, namely:

- a. The reflector with maximum value of \min_a , selected from among the 10 reflectors in Section 4.1.c.
- b. The reflector with d as small as possible when $\min_a > 2.5$, selected from among the 10 reflectors in Section 4.2.c.

For these two reflectors it is investigated whether or not it is possible to enhance \min_a and decrease d , respectively, by changing the lengths of the transmission lines and the spacings between the dipoles. The changes are made in such a way that lengths and spacings are varied independently. That is, it is permitted for the transmission lines to be of unequal lengths or the dipoles to be non-equispaced and the reflector to be non-linear.

Again, let l_1 and l_2 be the lengths of the transmission lines and let n_{11} , n_{21} , n_{31} , n_{41} and n_{12} , n_{22} , n_{32} , n_{42} be the abscissas and ordinates of the dipoles 1, 2, 3, and 4, respectively (see Fig. 28). First, with the previously obtained values of the coordinates \min_a (d) is increased (decreased) by increasing or decreasing l_1 . With the value obtained for l_1 , we then change l_2 . Next, with the original values of the lengths, we change n_{11} , n_{21} , n_{31} , n_{41} , n_{12} , n_{22} , n_{32} , and n_{42} in turn in an attempt to make \min_a larger and larger (d smaller and smaller). l 's and n 's are changed in steps of 0.01λ .

4.4.a. Maximum of \min_a

In Table 7 we have shown the results obtained for the reflector with the maximum value of \min_a . It appears that changes in l_1 and l_2 do not increase \min_a . By varying the positions of the dipoles, it is seen that \min_a is increased by changing n_{11} , n_{21} , and n_{31} . Only n_{21} has been considerably changed. \min_a is increased by about 4 % and at the same time d is increased by about 10 %. As might be expected, it is found that, if coupling is neglected, then it is not possible to increase \min_a by performing the changes described above.

4.4.b. d small and $\min_a > 2.5$

For the reflector with d as small as possible when $\min_a > 2.5$, the results are shown in Table 8. We see that only a small decrement in d is obtainable. When the lengths are changed, l_1 is decreased by 0.01λ and l_2 is increased by 0.01λ . When the positions of the dipoles are varied n_{31} is changed by 0.01λ and the other coordinates are not changed. We notice that after making the changes

in the lengths, \min_a is decreased and after making the changes in the coordinates \min_a is increased, but in both cases by small amounts.

The two examples dealt with above show that $\min_a(d)$ may only be increased (decreased) by small amounts by permitting the transmission lines to be of unequal length, the dipoles to be non-equispaced and the reflector to be non-linear. Furthermore, as might be expected from the basic idea of the Van Atta reflector, $\min_a(d)$ is usually decreased (increased) when the lengths are unequal and when the dipoles are not placed symmetrical about a center.

5. CONCLUSION

In this report has been investigated the manner in which the reradiation of a Van Atta reflector back in the direction of incidence depends on the length (l) of the transmission lines, the characteristic impedance (Z_0) of the lines, the imaginary part (X_{An}) of the antenna impedance, and the spacing (a) between the dipoles. Only interspacings smaller than three wavelengths have been considered.

A method has been presented by means of which the deviation from Van Atta effect (d) is evaluated from knowledge of the currents generated in the dipoles by an incident wave. It is shown that, when we wish to compare the back-scattering of two reflectors with different parameters, the method of evaluating the deviation from Van Atta effect is as good as a direct comparison between the reradiation patterns.

A reflector with Van Atta effect has not been found. However, when coupling is neglected, it is found that, for given values of l and Z_0 , the reflector will have a smaller deviation from Van Atta effect if X_{An} is chosen so that $X_{An} + Z_0 \cot \beta l = 0$, than if this is not the case. Furthermore, the Van Atta effect may be increased as much as we desire by increasing $|Z_0 / \sin \beta l|$ relative to $|R_{An} + i(X_{An} + Z_0 \cot \beta l)|$. However, at the same time the reradiation decreases.

When l , X_{An} , and Z_0 are equal to $0.25\lambda + j0.50\lambda$, 0 ohms, and R_{An} , respectively, \min_a (the minimum value of the back-scattering as a function of the angle of incidence), as a function of l , X_{An} , and Z_0 , has the flattest maximum. This is valid for all $a > \lambda/8$ and it is found that $\max_a / \min_a = \sqrt{2}$ (\max_a is the maximum value of the back-scattering as a function of the angle of incidence). For $a < 3\lambda$ it has been found that, when l , X_{An} , and Z_0 have the values given above, the deviation from Van Atta effect is smallest for $a = 1.5\lambda$. Furthermore, it has been found that if we desire a spacing less than 1.5λ for which d is small, we should choose $a = 0.50, 0.75, 0.25, 1.00$, or 1.25λ . These distances are mentioned in order of increasing d .

When a tends to zero then \min_a tends to \max_a .

When coupling is taken into account, some of the results described above are altered. It turns out that coupling for specific values of l , a , X_{An} , and Z_0 may have a large influence. Coupling usually causes \min_a to decrease and d to increase. For $a < 0.25\lambda$, \min_a is only half of its maximum value. For some values of the parameters and for some angles of incidence the back-scattering is increased 50 % by coupling.

The analysis of the reradiation from 1600 sets of parameters has indicated that variations in \min_a and d as functions of the parameters of the reflector are not affected by coupling. That is, including coupling does change the values of \min_a and d but the variations in these values appear to be the same as those described above where coupling was neglected.

By using the results from the investigation in which coupling is neglected, and the results of the calculations for the 1600 sets of parameters, the reradiation pattern has been optimized by using two different criteria. First, the reflector for which \min_a is as large as possible, and d as small as possible has been found. Next, there has been obtained values of the parameters of some reflectors for which \min_a is above various prescribed levels, and d is as small as possible.

By using the first criterion, a reflector has been found for which coupling increases \min_a over the maximum value which is obtainable if coupling were not present. However, the increment is negligible.

By using the second criterion it turns out that, when we require a larger value of \min_a , then we obtain a larger value of d . In this case it has not been possible to find values of l , a , X_{An} , and Z_0 for which coupling decreases the deviation from Van Atta effect so that it is smaller than that which may be obtained if coupling were not present.

For both types of reflectors obtained by the optimization it turns out that a is close to 1.50λ . For the first type of reflector and for the second type with $\min_a > 2.5$, it has been found that l , X_{An} , and Z_0 are close to the optimum values found when coupling is neglected. For the second type it is found that when we prescribe \min_a to be above a level smaller than 2.5, d is usually decreased by increasing Z_0 .

A further optimization shows that, due to coupling, \min_a may be increased and d decreased by permitting the transmission lines to be of unequal lengths and the dipoles to be placed asymmetrically about the center of the reflector. However, the increment in \min_a (the decrement in d) is at best small and most asymmetries would in fact have the opposite effect, that is, \min_a would be decreased and d increased.

6. LITERATURE

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l/λ	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8
a/λ	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8
X_{An}/ohms	-42	-20	0	20	42			
Z_0/ohms	50	60	73	80	90			

Table 1.
Combinations of the parameters.

Set		d	\min_a	l/λ	a/λ	X_{An}/ohms	Z_0/ohms
1	b	3.42	2.618	0.750	1.000	0	60
	a	3.39	2.630	0.750	1.000	- 5	60
2	b	3.21	2.598	0.250	1.000	20	90
	a	3.25	2.616	0.250	1.000	25	90
3	b	2.81	2.595	0.750	1.000	0	73
	a	3.26	2.638	0.750	1.000	0	63
4	b	2.98	2.588	0.625	1.000	- 43	50
	a	3.26	2.632	0.645	1.000	- 38	50
5	b	3.36	2.566	0.125	1.000	- 43	60
	a	3.00	2.602	0.105	1.000	- 53	60
6	b	2.89	2.566	0.125	1.000	- 43	73
	a	3.57	2.602	0.145	0.980	- 43	68
7	b	3.56	2.544	0.250	1.000	20	80
	a	3.25	2.616	0.250	1.000	25	90
8	b	2.57	2.529	0.750	1.000	0	80
	a	3.38	2.637	0.730	1.000	- 10	60
9	b	3.11	2.528	0.750	1.000	20	73
	a	3.15	2.641	0.790	1.000	15	63
10	b	2.70	2.509	0.125	1.000	- 43	80
	a	3.03	2.584	0.145	1.000	- 43	75
11	b	2.25	2.619	0.250	1.500	0	73
	a	2.53	2.820	0.230	1.540	- 5	68
12	b	2.49	2.744	0.750	1.500	0	73
	a	2.50	2.777	0.750	1.520	0	73

Table 2.

Maximum of \min_a .

Fig.	Coupling	d	\min_a	\max_a	g_a	d_s	l/λ	a/λ	X_{An}/ohms	Z_0/ohms
7.2	+	3.15	2.64	3.60	2.95	2.91	0.79	1.00	15	63
	-	3.32	2.73	4.12	3.35	2.72	0.79	1.00	15	63
7.3	+	2.53	2.82	3.67	3.13	2.39	0.23	1.54	- 5	68
	-	2.69	2.73	4.03	3.43	2.33	0.23	1.54	- 5	68
7.4	+	2.48	2.54	3.11	2.80	3.43	0.67	1.00	- 35	73
	-	2.70	2.71	3.74	3.16	3.36	0.67	1.00	- 35	73
7.5	+	1.67	2.48	3.11	2.78	3.21	0.25	1.50	0	93
	-	1.66	2.66	3.38	3.01	3.15	0.25	1.50	0	93

Table 3.

Characteristics for the Figs. 7.2 - 7.5.

Set		d	min _a	ℓ/λ	a/λ	X_{An}/ohms	Z_0/ohms	$X_{An}+Z_0\cot\beta\ell/\text{ohms}$
1	a	2.89	2.129	0.250	0.50	0	73	0
	b	2.23	1.909	0.330	0.50	15	73	- 25
	c	2.54	2.073	0.330	0.50	40	73	0
	d	1.17	1.413	0.330	0.50	95	173	0
2	a	2.81	2.595	0.750	1.00	0	73	0
	b	2.77	2.468	0.770	1.00	0	73	- 9
	c	2.79	2.589	0.770	1.00	9	73	0
	d	1.11	1.493	0.770	1.00	27	213	0
3	a	2.71	2.081	0.125	0.50	- 42	73	31
	b	2.23	2.034	0.185	0.52	- 42	73	- 10
	c	2.42	2.122	0.185	0.52	- 31	73	0
	d	1.10	1.466	0.185	0.52	- 75	173	0
4	a	2.48	2.334	0.625	1.00	- 42	73	31
	b	1.26	1.629	0.565	1.02	- 177	73	- 8
	c	1.26	1.672	0.565	1.02	- 169	73	0
	d	1.01	1.401	0.565	1.02	- 215	93	0
5	a	1.94	1.765	0.375	0.50	42	73	- 31
	b	1.35	1.675	0.415	0.52	117	73	- 6
	c	1.40	1.706	0.415	0.52	124	73	0
	d	1.13	1.494	0.415	0.52	157	93	0
6	a	2.28	2.045	0.875	1.00	42	73	31
	b	1.26	1.636	0.935	1.02	162	73	- 7
	c	1.26	1.672	0.935	1.02	169	73	0
	d	1.01	1.401	0.935	1.02	215	93	0
7	a	2.60	1.973	0.250	0.50	- 20	73	- 20
	b	2.43	2.062	0.230	0.52	- 20	73	- 11
	c	2.63	2.159	0.230	0.52	- 9	73	0
	d	1.07	1.438	0.230	0.52	- 24	193	0
8	a	2.89	2.332	0.750	1.00	- 20	73	- 20
	b	2.48	2.537	0.670	1.00	- 35	73	5
	c	2.47	2.497	0.670	1.00	- 40	73	0
	d	1.08	1.465	0.670	1.00	- 106	193	0
9	a	2.46	1.616	0.375	0.50	20	73	- 53
	b	1.57	1.779	0.395	0.52	80	73	- 14
	c	1.65	1.858	0.395	0.52	94	73	0
	d	1.12	1.486	0.395	0.52	146	113	0
10	a	2.65	1.875	0.875	1.00	20	73	- 53
	b	1.26	1.678	0.935	1.02	170	73	1
	c	1.26	1.672	0.935	1.02	169	73	0
	d	1.01	1.401	0.935	1.02	215	93	0
11	a	2.25	2.619	0.250	1.50	0	73	0
	b	2.10	2.550	0.270	1.52	0	73	- 9
	c	2.12	2.667	0.270	1.52	9	73	0
	d	0.65	1.451	0.270	1.52	29	233	0
12	a	2.49	2.744	0.750	1.50	0	73	0
	b	2.45	2.688	0.770	1.50	0	73	- 9
	c	2.47	2.743	0.770	1.50	9	73	0
	d	0.71	1.432	0.770	1.50	32	253	0

Table 4.

Optimization of d.

Set	d	\min_a	l/λ	a/λ	X_{An}/ohms	Z_0/ohms	$X_{An} + Z_0 \cot \beta l / \text{ohms}$
8	2.48	2.537	0.670	1.00	- 35	73	- 5
6	1.59	2.003	0.915	1.02	132	73	9
7	1.17	1.536	0.230	0.52	- 22	173	0
12	1.92	2.635	0.770	1.50	12	93	0
11	1.15	2.122	0.270	1.52	16	133	0
11	0.71	1.555	0.270	1.52	26	213	0

Table 5.

The smallest values of d depending
on \min_a . Coupling taken into account.

d	\min_a	l/λ	a/λ	X_{An}/ohms	Z_0/ohms	$X_{An} + Z_0 \cot \beta l / \text{ohms}$
1.66	2.659	0.25	1.50	0	93	0
1.03	2.128	0.25	1.50	0	153	0
0.67	1.563	0.25	1.50	0	233	0

Table 6.

The smallest values of d depending
on \min_a . Coupling neglected.

	d	\min_a	l_1/λ	l_2/λ	n_{11}/λ	n_{21}/λ	n_{31}/λ	n_{41}/λ	n_{12}/λ	n_{22}/λ	n_{32}/λ	n_{42}/λ
before changes	3.15	2.641	0.79	0.79	0.00	1.00	2.00	3.00	0.00	0.00	0.00	0.00
after changes in l 's	3.15	2.641	0.79	0.79	0.00	1.00	2.00	3.00	0.00	0.00	0.00	0.00
after changes in n 's	3.43	2.719	0.79	0.79	0.01	0.94	2.02	3.00	0.00	0.00	0.00	0.00

Table 7.

Maximizing of \min_a by changing the lengths and the spacings independently. $X_{An} = 15$ ohms, $Z_0 = 63$ ohms.

	d	\min_a	l_1/λ	l_2/λ	n_{11}/λ	n_{21}/λ	n_{31}/λ	n_{41}/λ	n_{12}/λ	n_{22}/λ	n_{32}/λ	n_{42}/λ
before changes	2.48	2.537	0.67	0.67	0.00	1.00	2.00	3.00	0.00	0.00	0.00	0.00
after changes in l 's	2.46	2.498	0.66	0.68	0.00	1.00	2.00	3.00	0.00	0.00	0.00	0.00
after changes in n 's	2.45	2.555	0.67	0.67	0.00	1.00	1.99	3.00	0.00	0.00	0.00	0.00

Table 8.

Decreasing d by changing the lengths and the spacings independently. $X_{An} = -35$ ohms, $Z_0 = 73$ ohms.

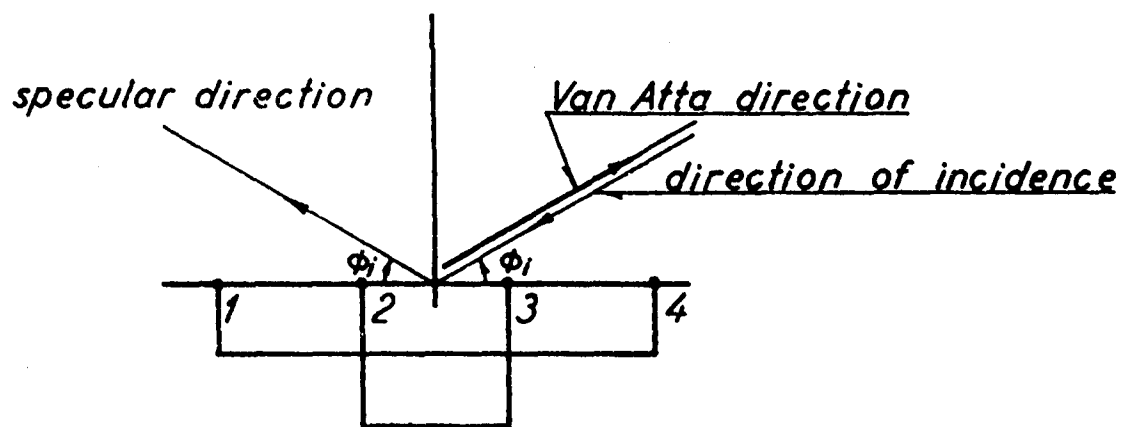


Fig.1. The reflector.

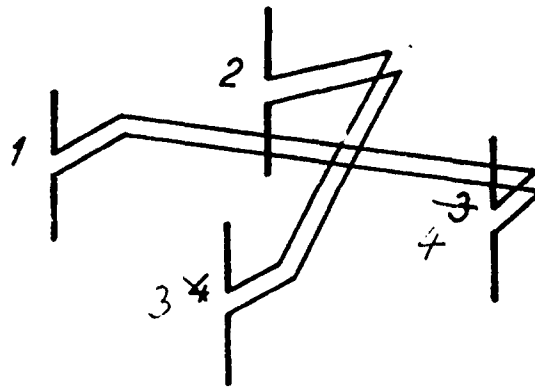


Fig. 2. The reflector.

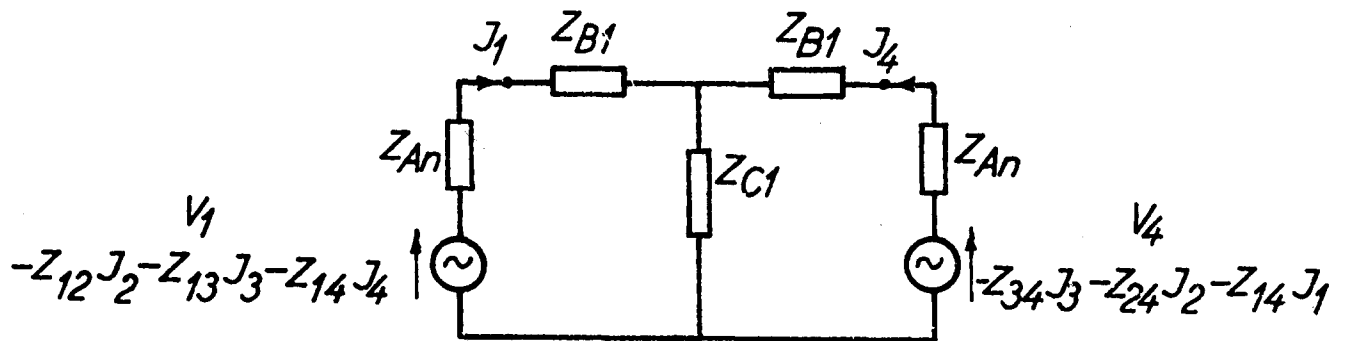


Fig. 3. Equivalent circuit for dipoles 1 and 4.

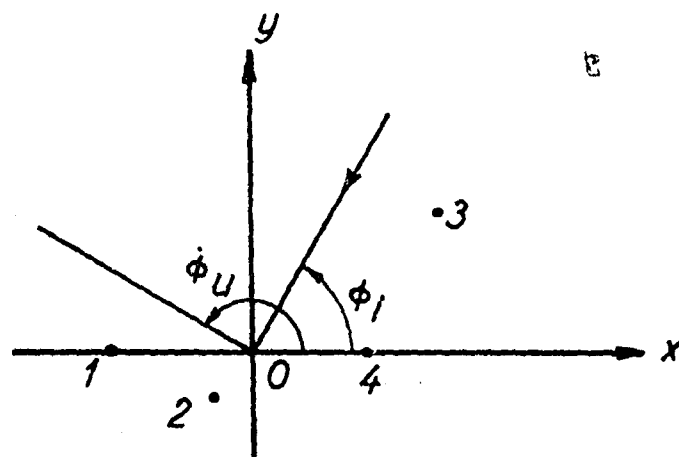


Fig. 4. Placing of dipoles.

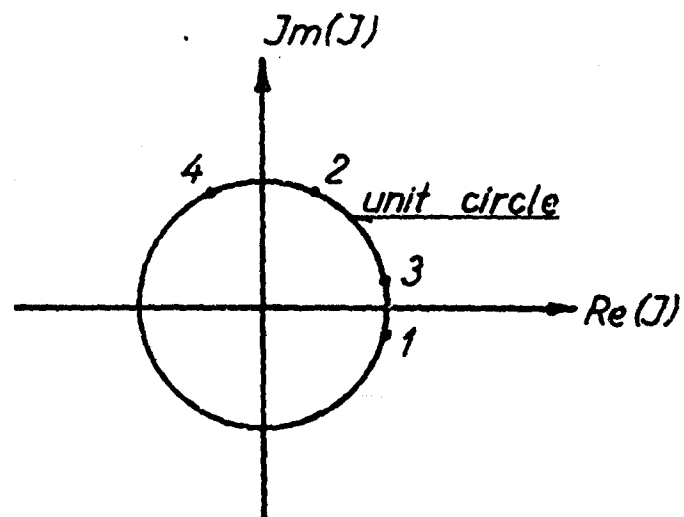


Fig. 5. Current plane.

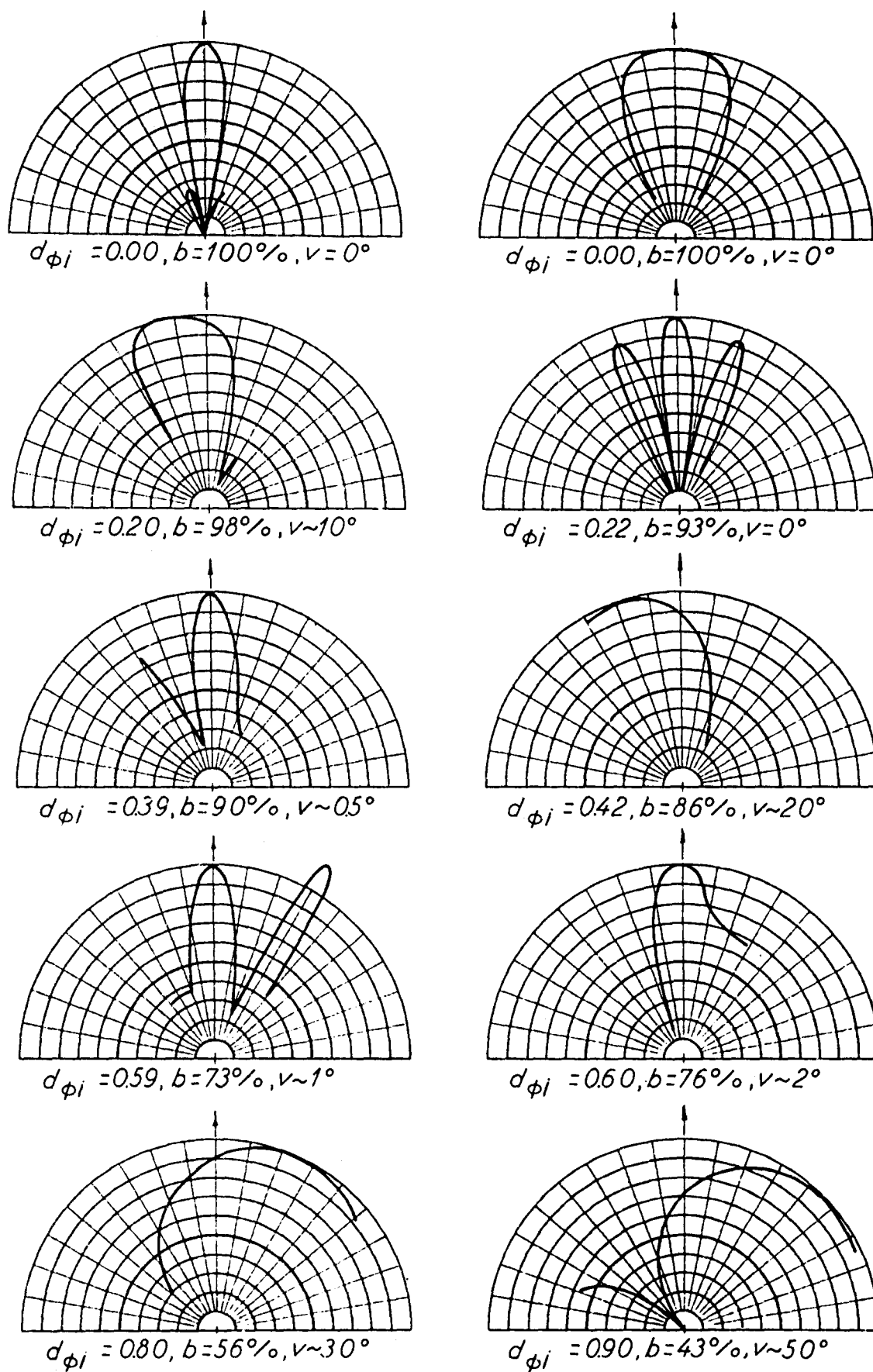


Fig. 6. The beam in the Van Atta direction for different values of d_{ϕ_i} .

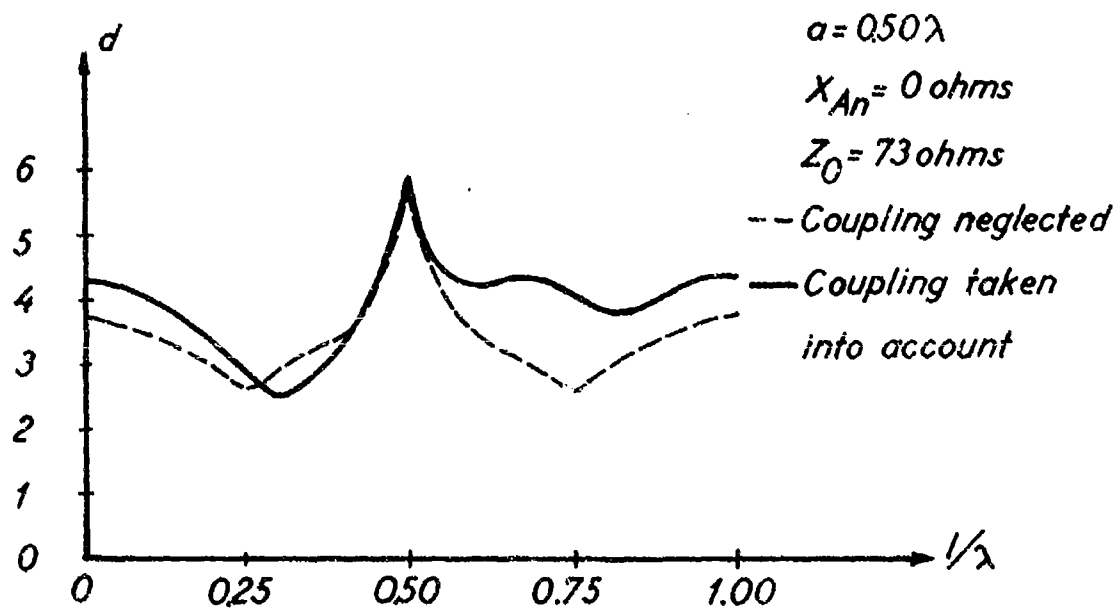


Fig. 7. d as a function of the length of the transmission lines.

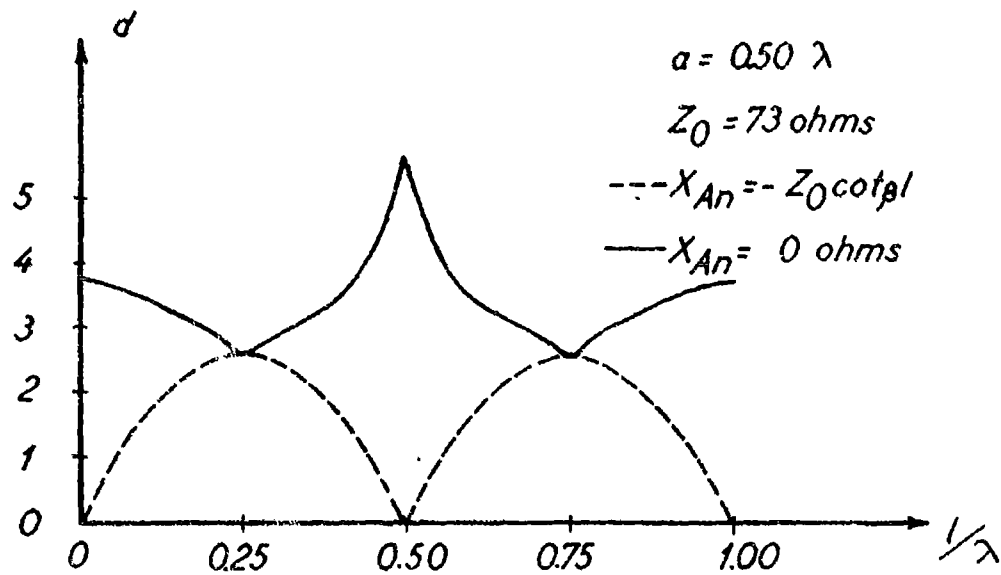


Fig. 8. d as a function of l when $X_{An} = -Z_0 \cot \beta l$ and $X_{An} = 0 \text{ ohms}$. Coupling neglected.

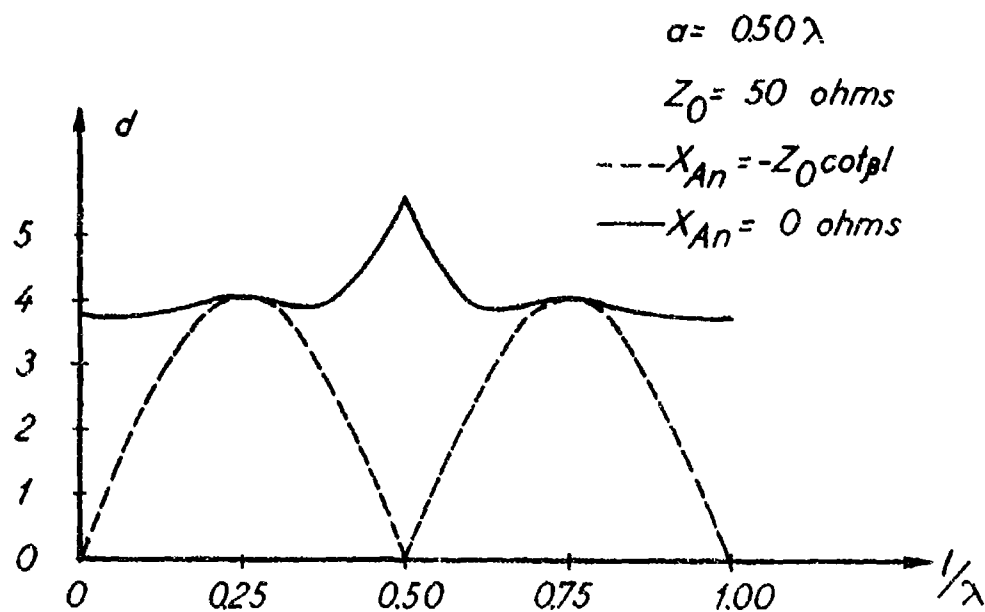


Fig. 9. d as a function of l when $X_{An} = -Z_0 \cot \beta l$ and $X_{An} = 0 \text{ ohms}$. Coupling neglected.

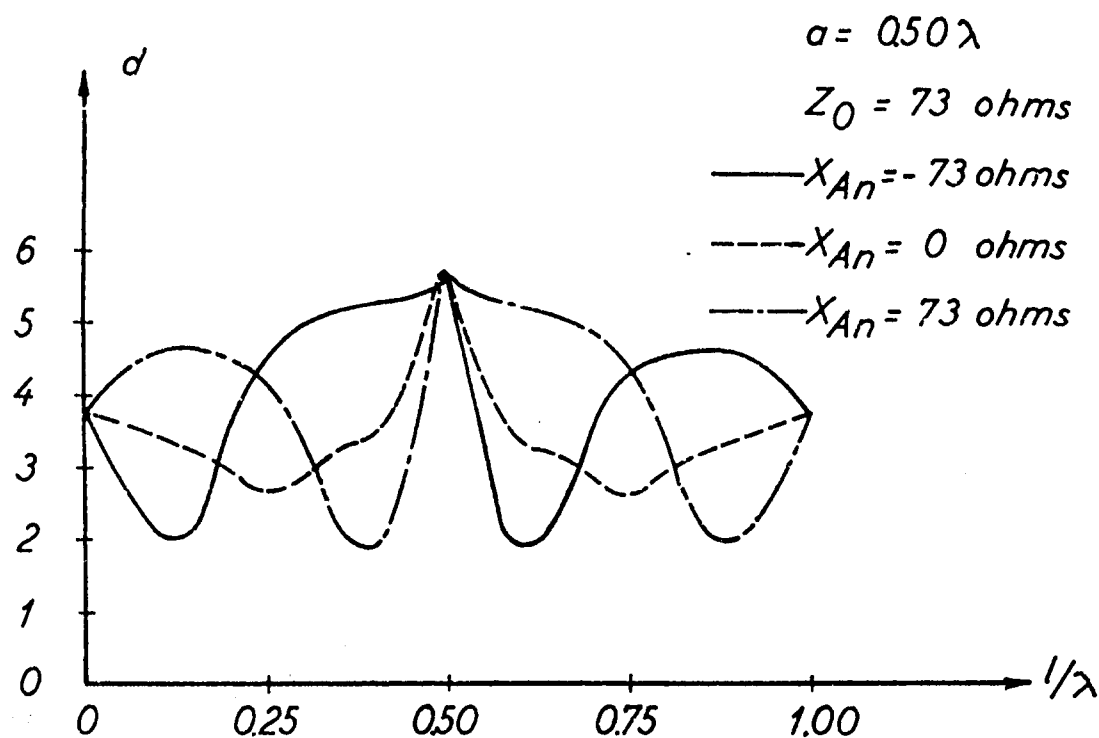


Fig. 10. d as a function of the length for $X_{An} = -73, 0$, and 73 ohms . Coupling neglected.

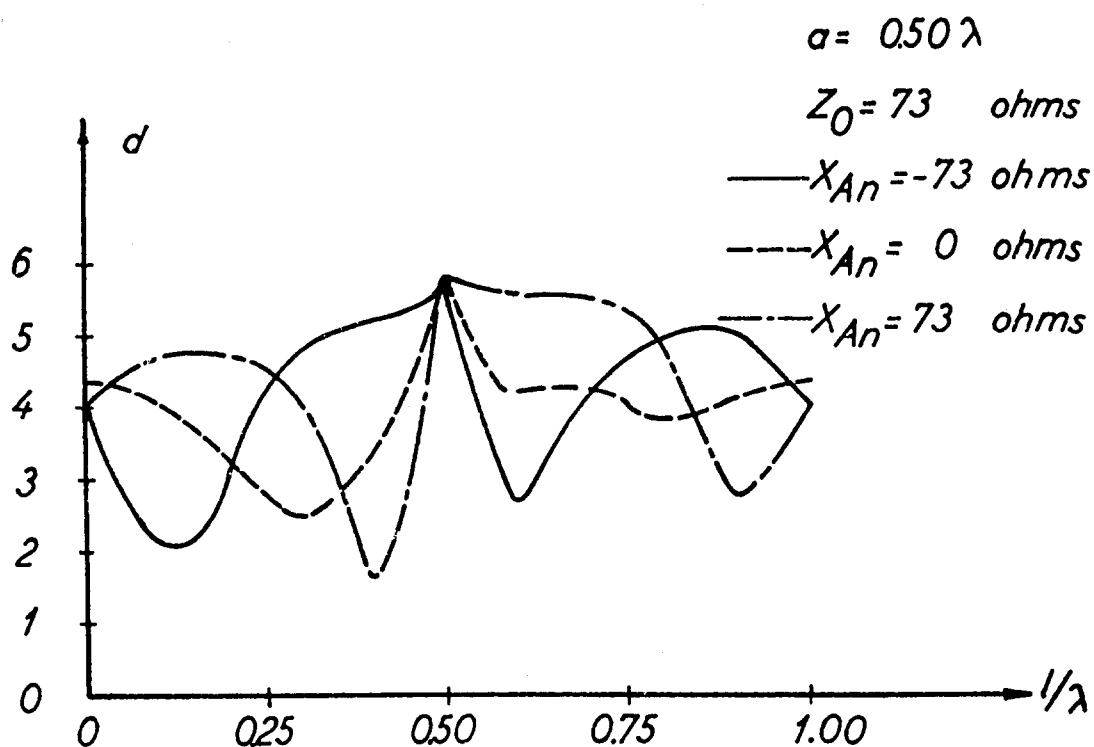


Fig. 11. d as a function of the length for $X_{An} = -73, 0$, and 73 ohms . Coupling taken into account.

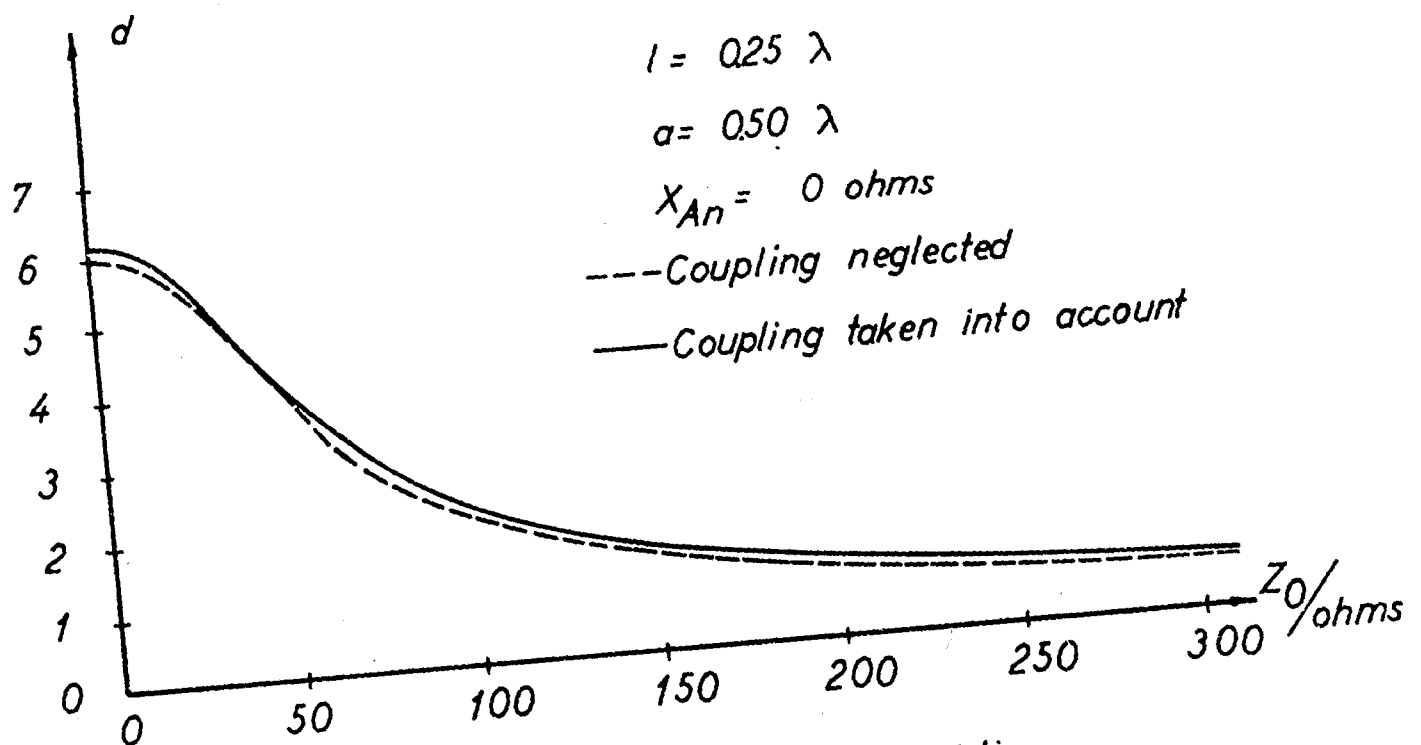


Fig. 12. d as a function of the characteristic impedance.

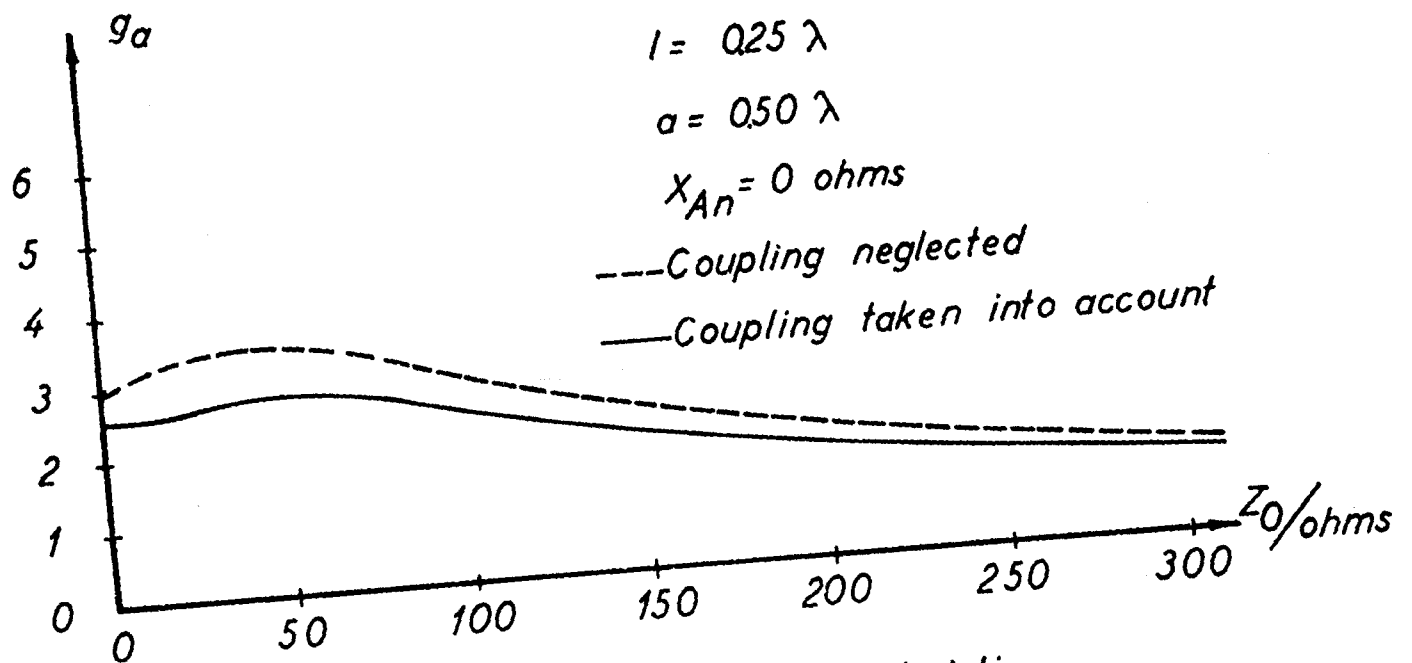


Fig. 13.a. g_a as a function of the characteristic impedance.

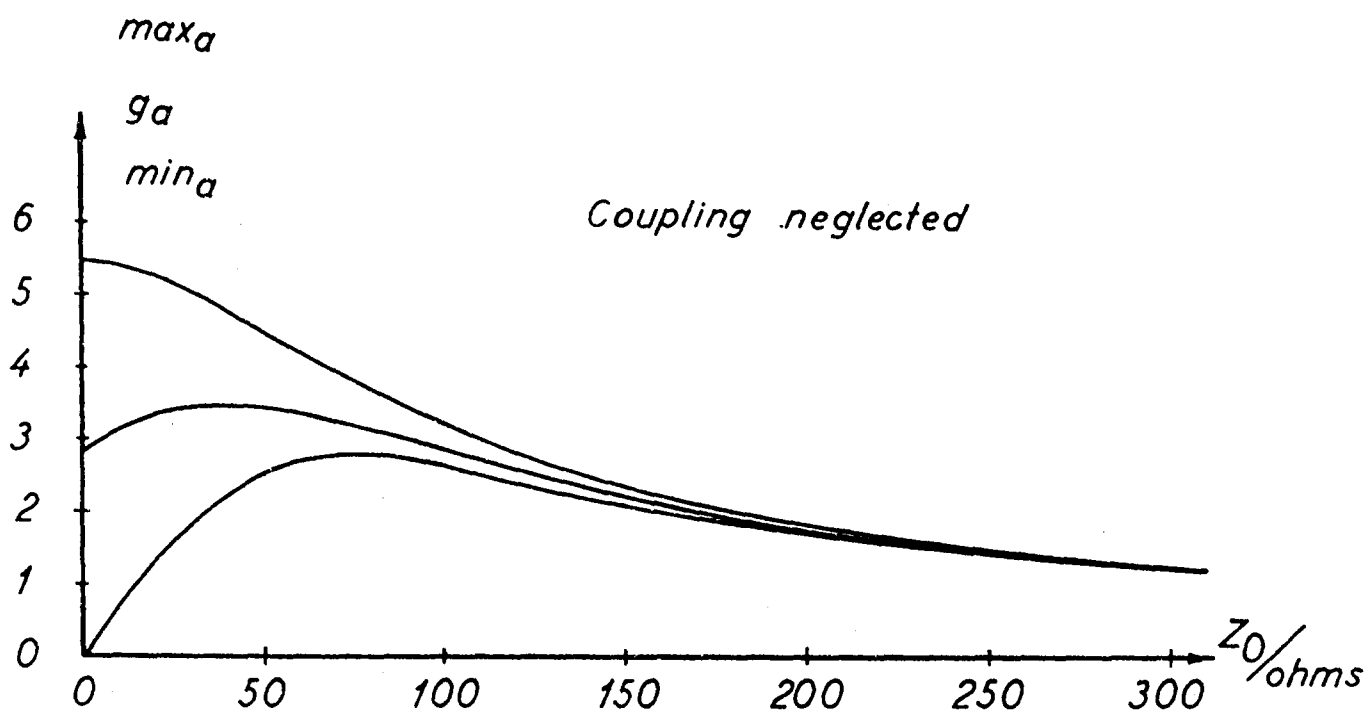


Fig. 13.b. max_a, g_a, min_a as a function of Z_0 .

Parameters as in Fig. 13.a.

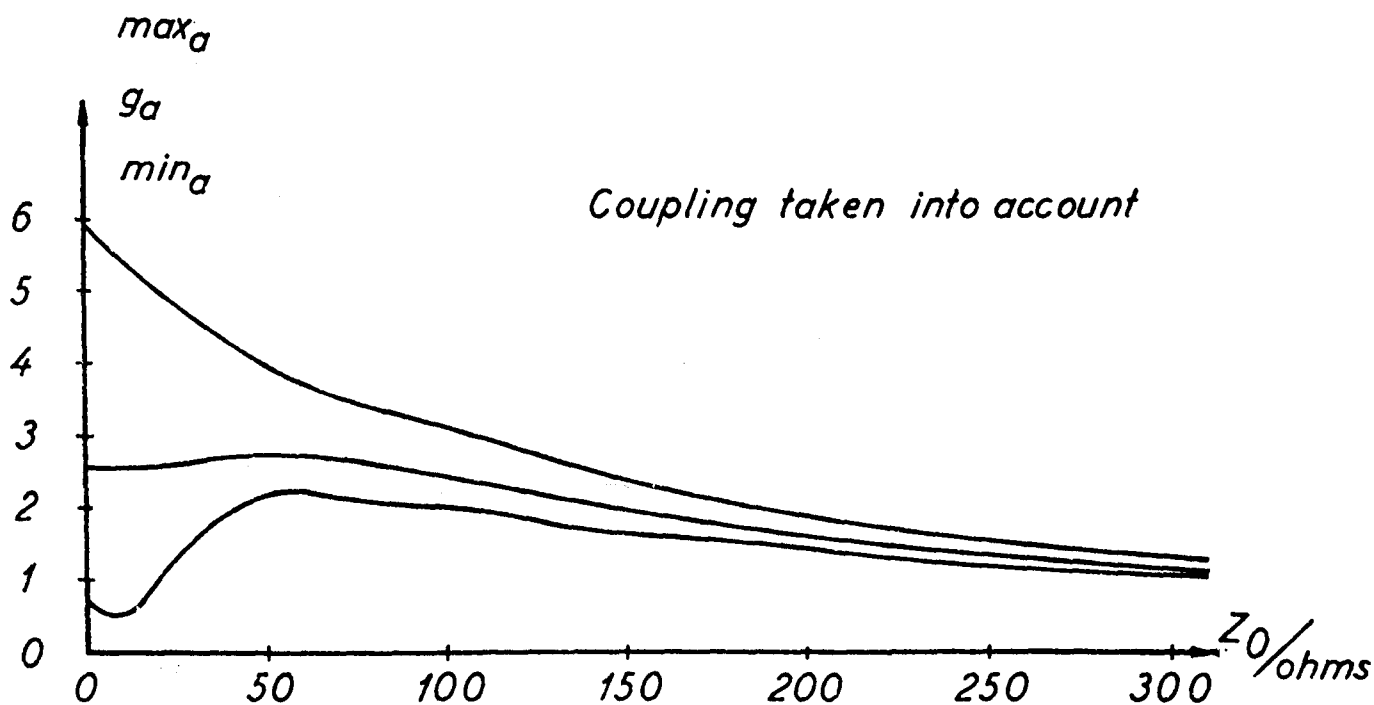


Fig. 13.c. max_a, g_a, min_a as a function of Z_0 .

Parameters as in Fig. 13.a.

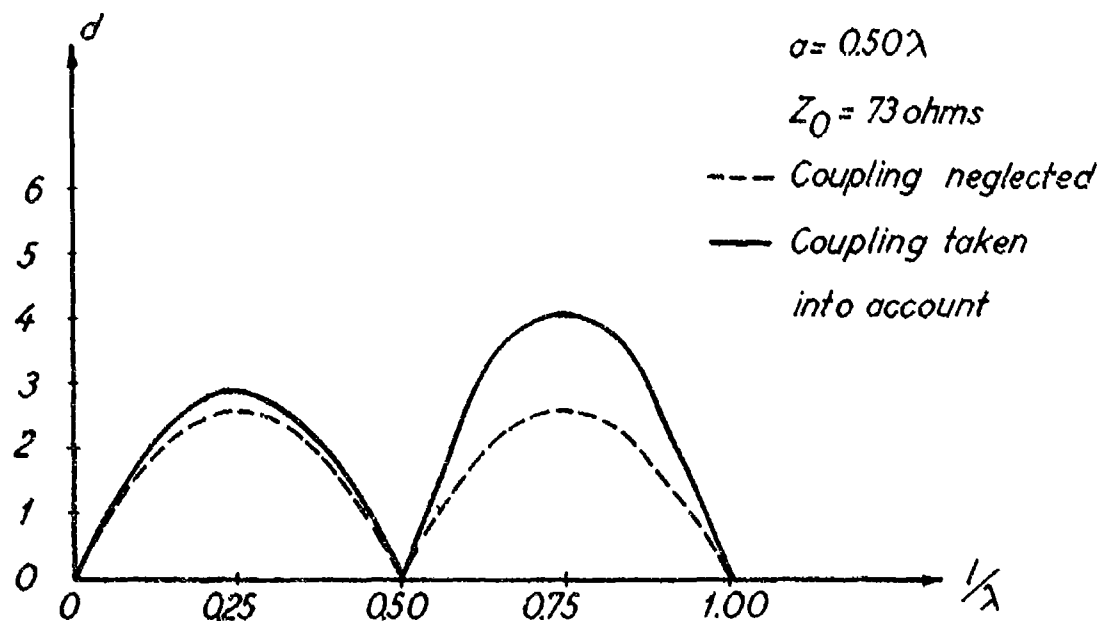


Fig. 14. d as a function of the length of the transmission lines. $X_{An} = -Z_0 \cot \beta l$ for each value of l .

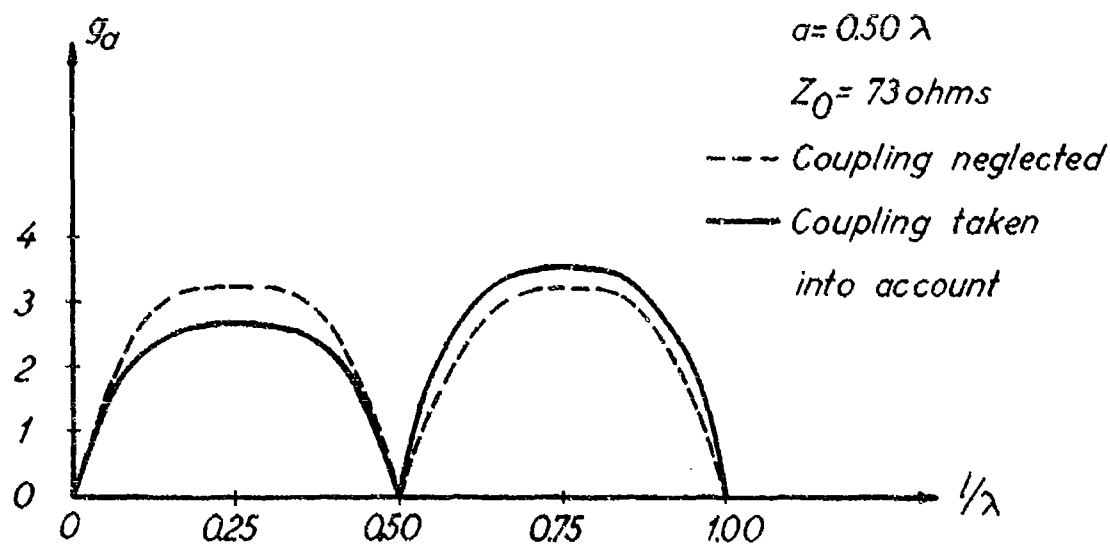


Fig. 15.a. g_a as a function of the length of the transmission lines. $X_{An} = -Z_0 \cot \beta l$ for each value of l .

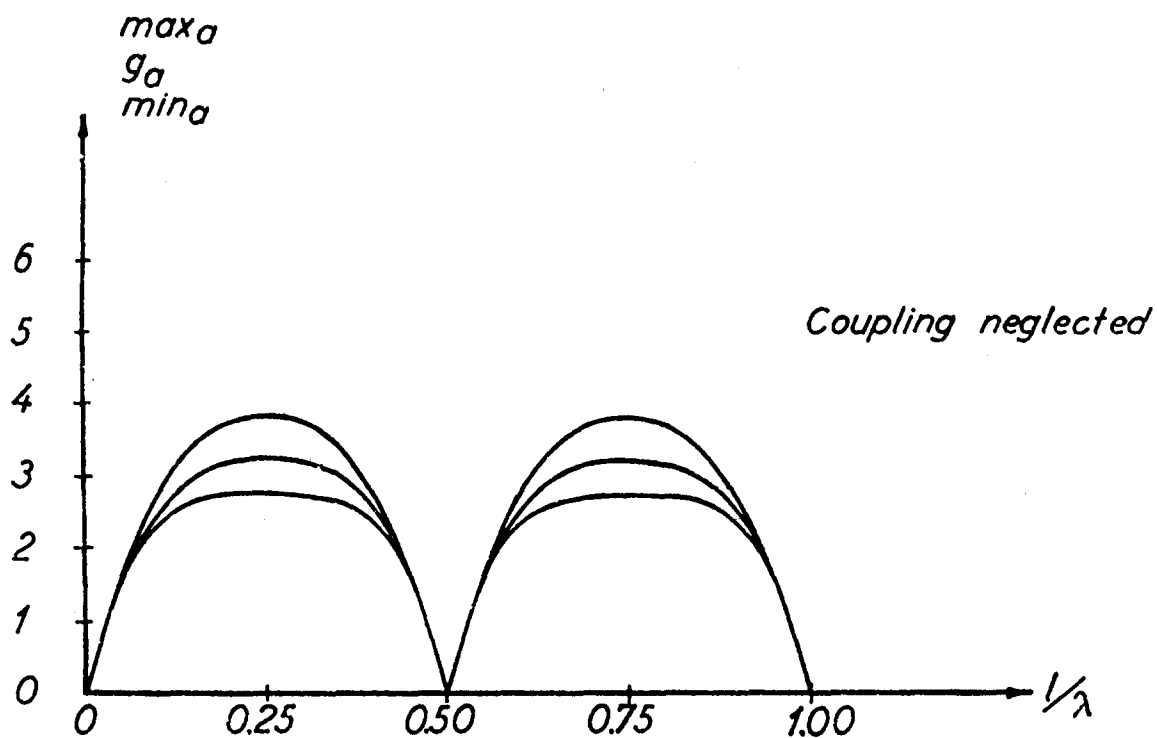


Fig. 15.b. max_a, g_a, min_a as a function of the length of the the transmission lines.
Parameters as in Fig. 15.a.

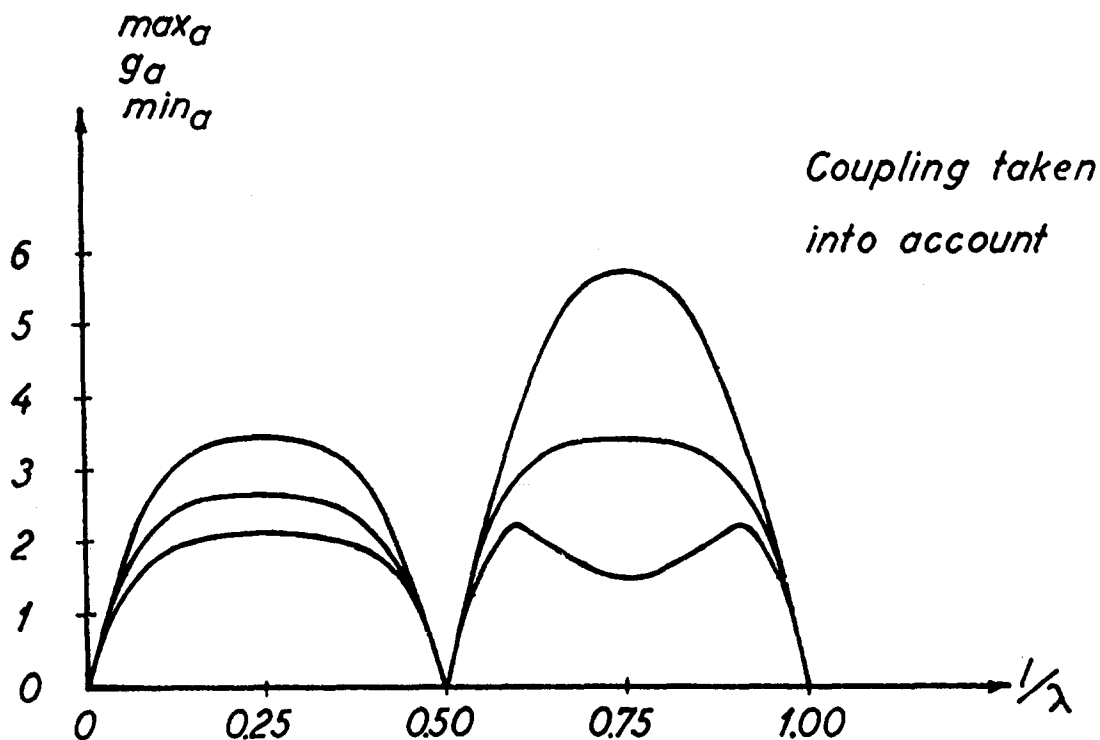


Fig. 15.c. max_a, g_a, min_a as a function of the length of the transmission lines.
Parameters as in Fig. 15.a.

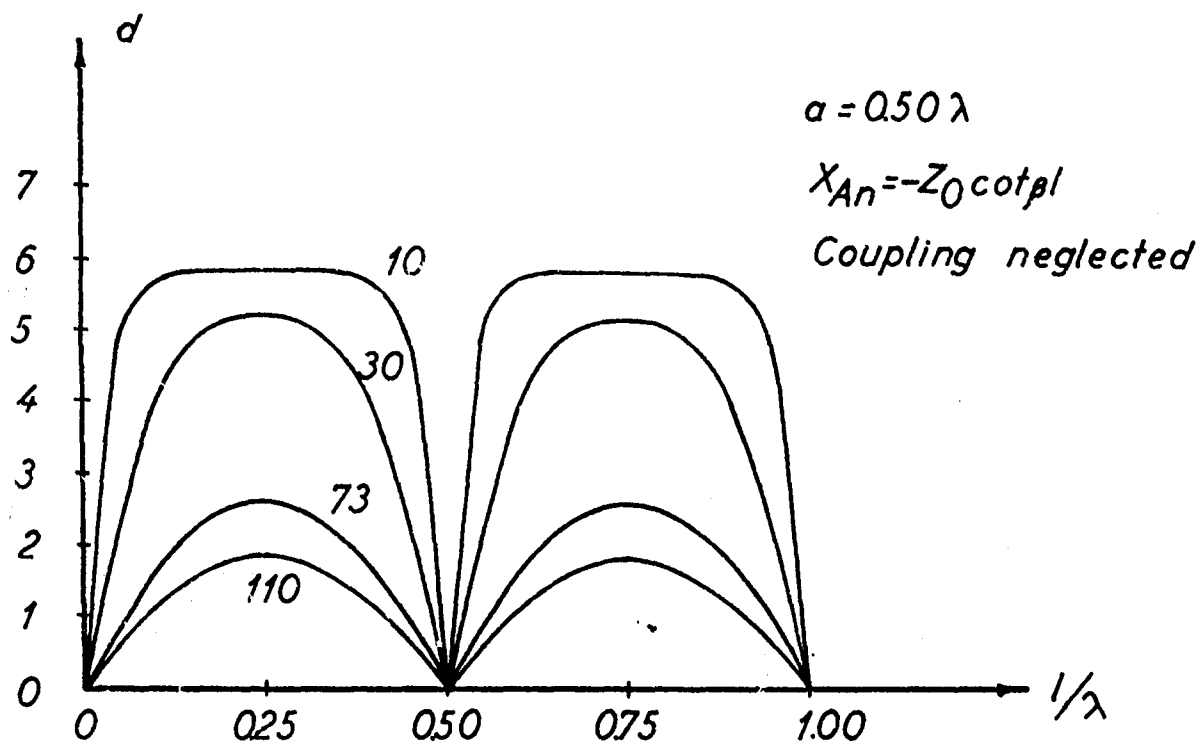


Fig. 16. d as a function of l for $Z_0 = 10, 30, 73$, and 110 ohms. The numbers on the curves indicate Z_0 in ohms.

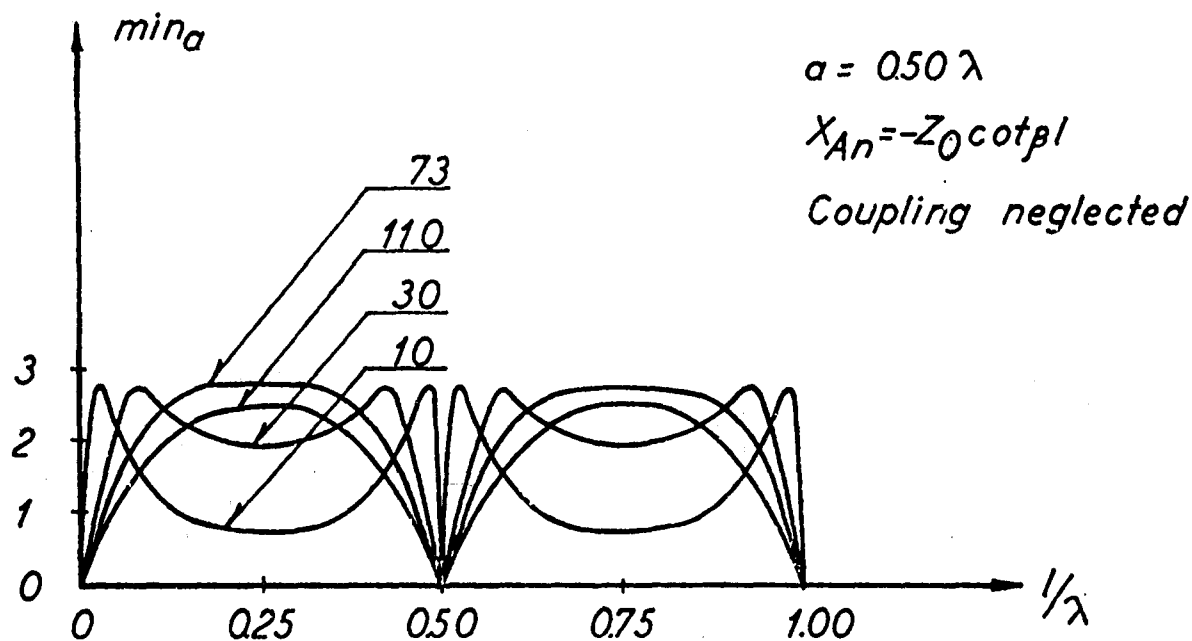


Fig. 17. \min_d as a function of l for $Z_0 = 10, 30, 73$, and 110 ohms. The numbers on the curves indicate Z_0 in ohms.

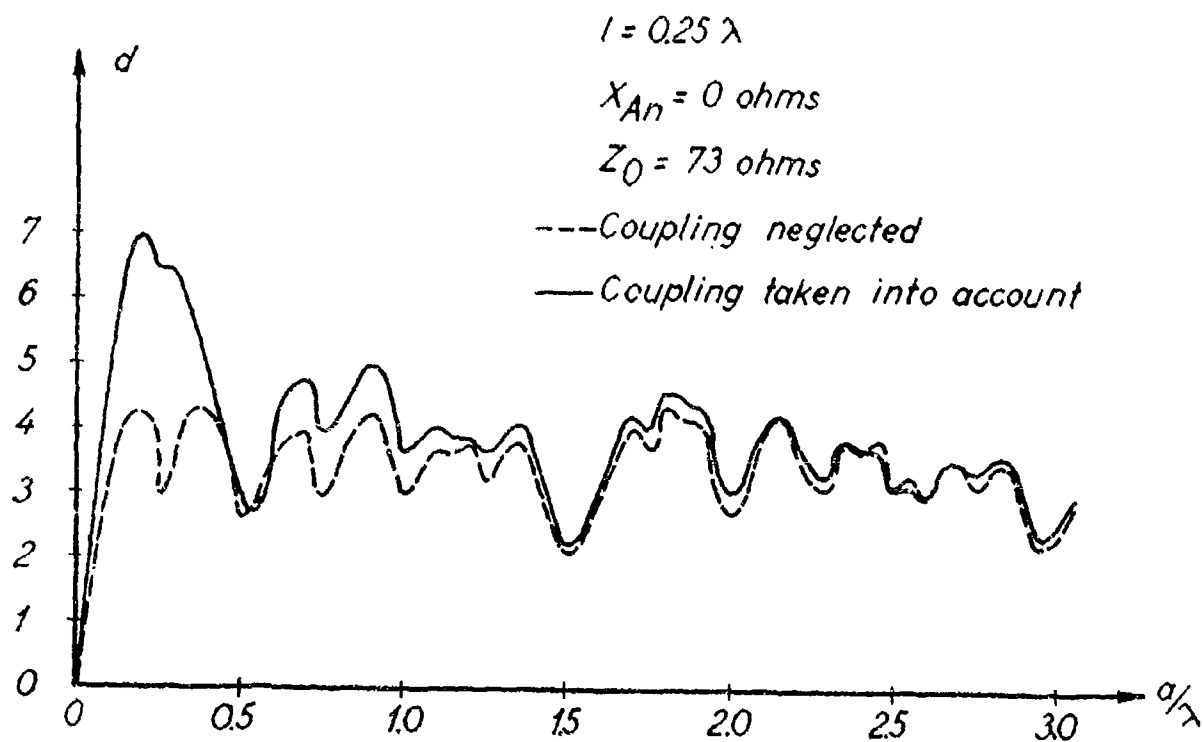


Fig.18. d as a function of the spacing.

$$X_{An} + Z_0 \cot \beta l = 0.$$

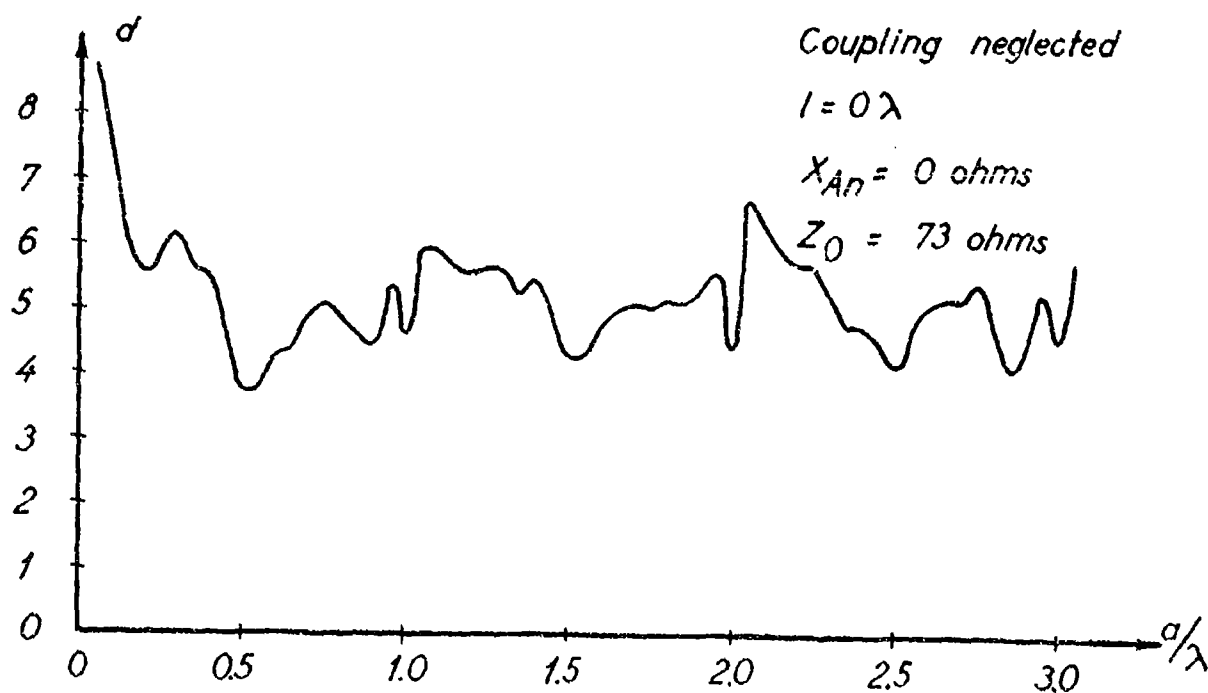


Fig.19. d as a function of the spacing.

$$X_{An} + Z_0 \cot \beta l \neq 0.$$

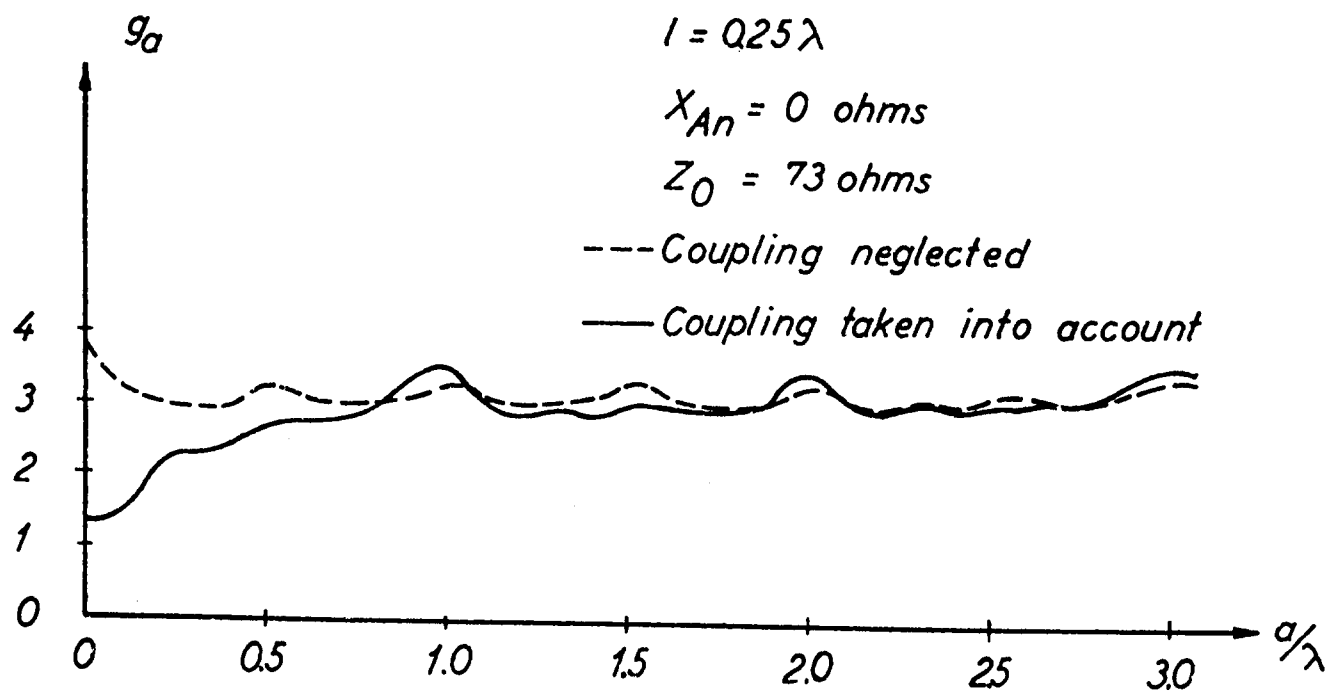


Fig. 20.a. g_a as a function of the spacing.

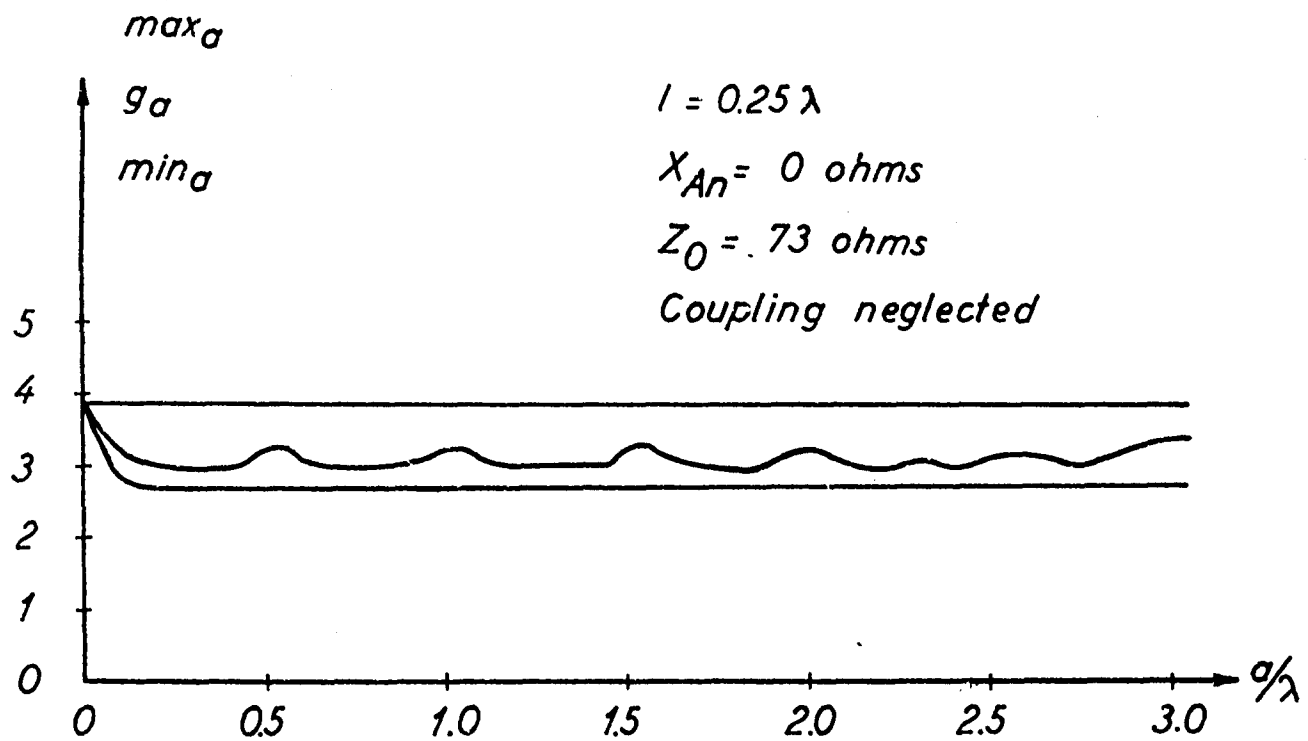


Fig. 20.b. max_a , g_a , and min_a as a function of the spacing.

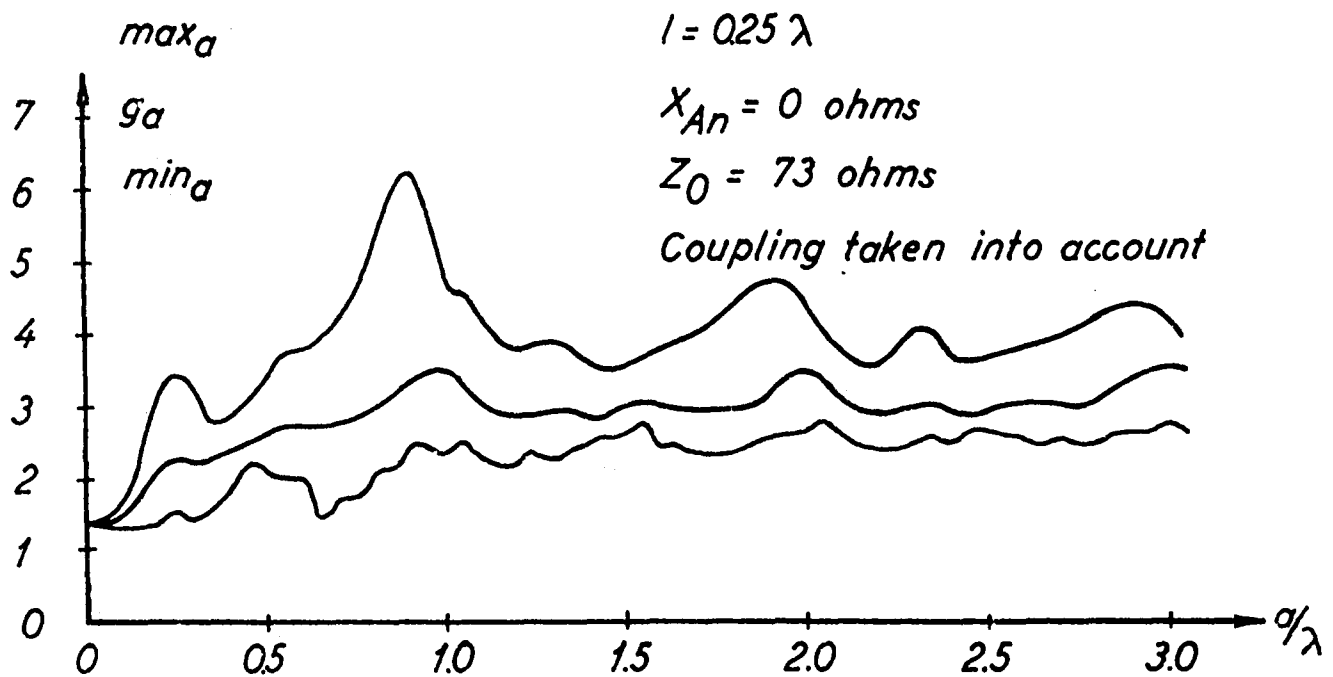


Fig. 20.c. max_a , g_a , and min_a as a function of the spacing.

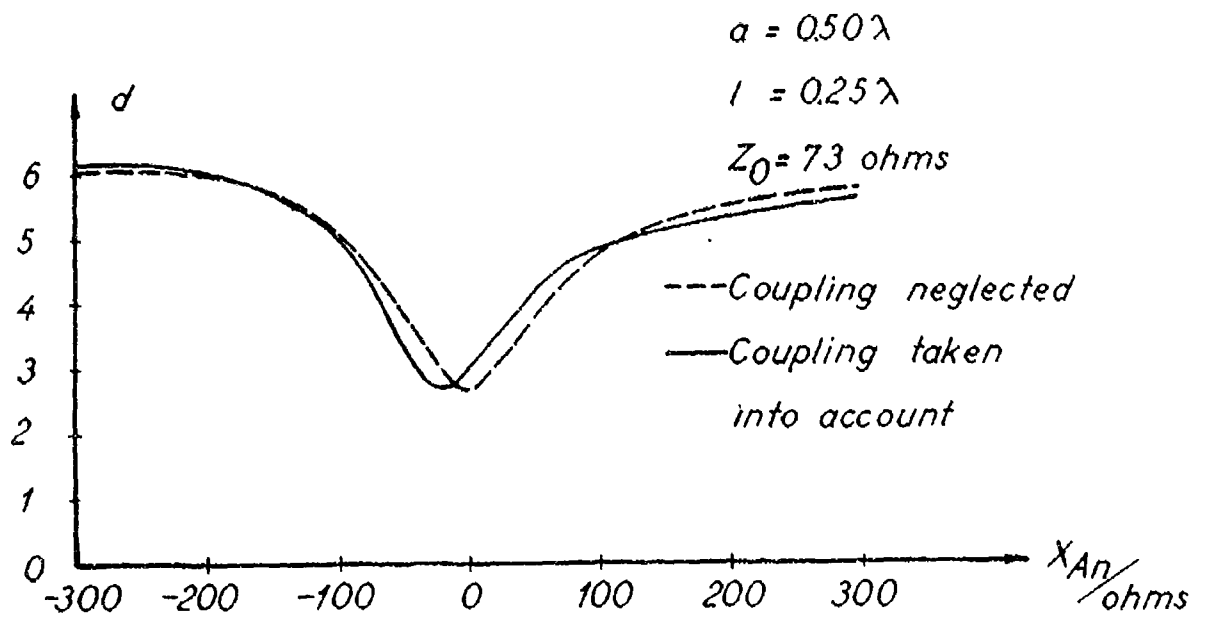


Fig. 21. d as a function of X_{An} .

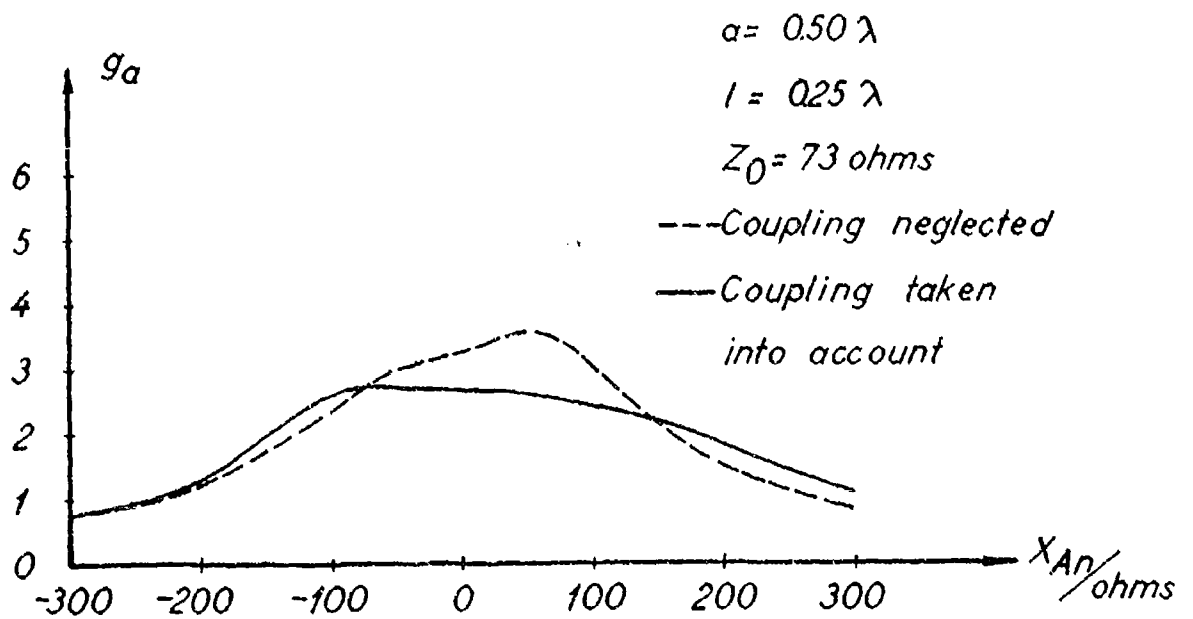


Fig. 22. g_a as a function of X_{An} .

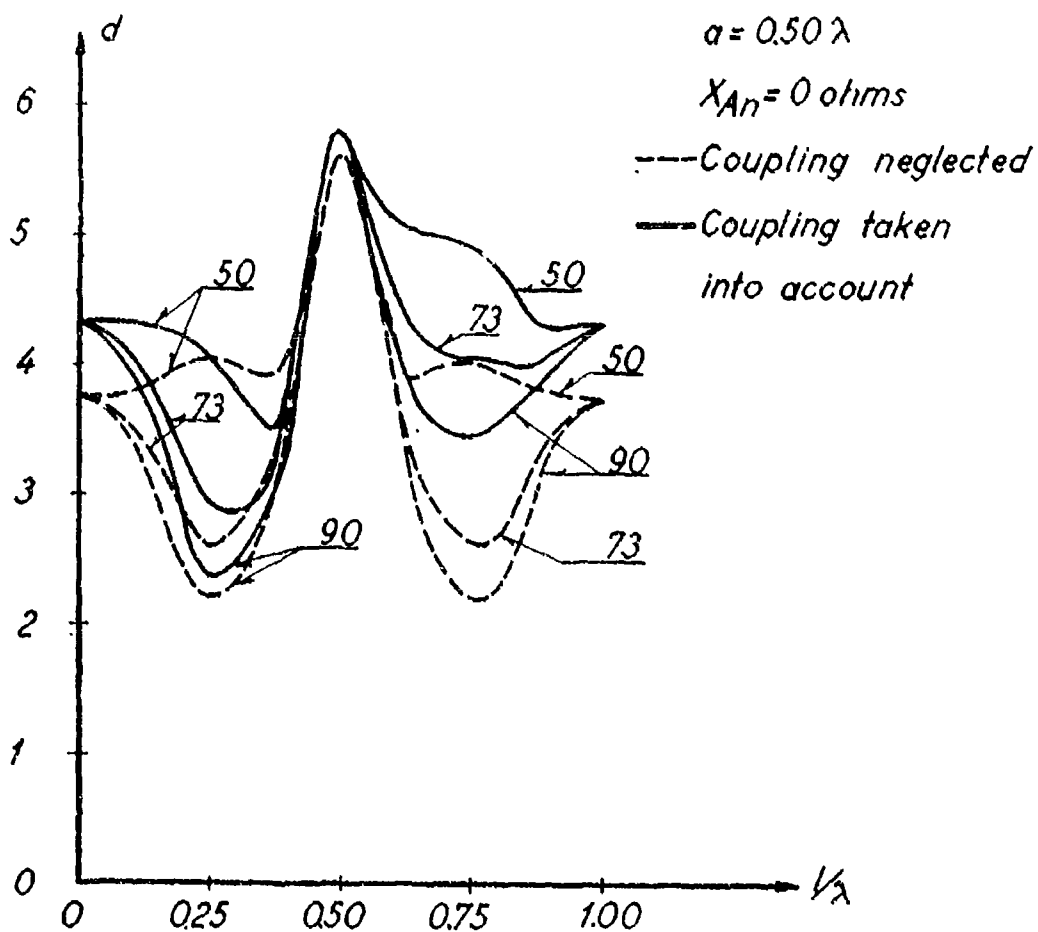


Fig. 23. d as a function of the length of the transmission lines for $Z_0 = 50, 73$, and 90 ohms. The numbers on the curves indicate Z_0 in ohms.

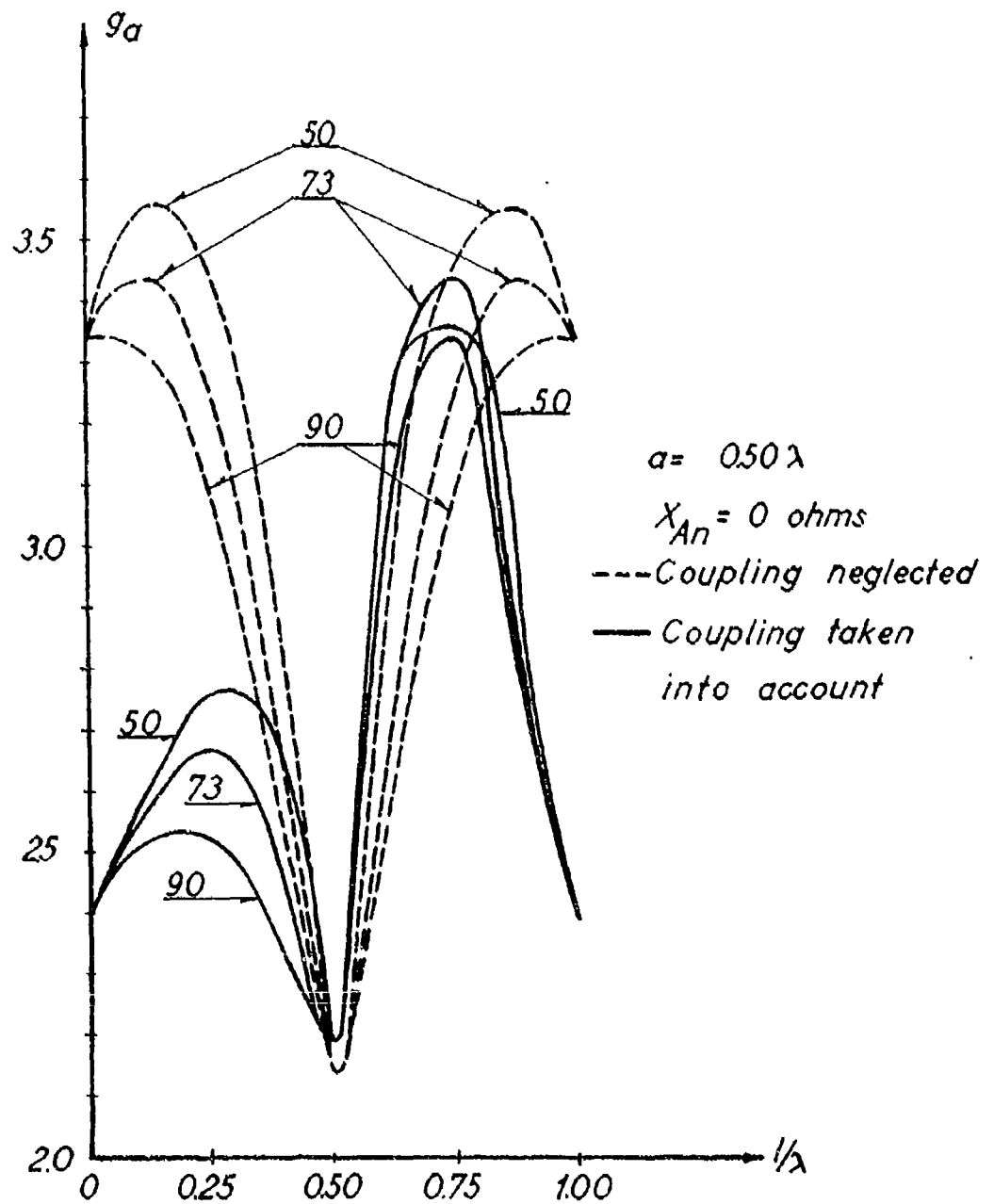


Fig.24. g_a as a function of the length of the transmission lines for $Z_0 = 50, 73$, and 90 ohms. The numbers on the curves indicate Z_0 in ohms.

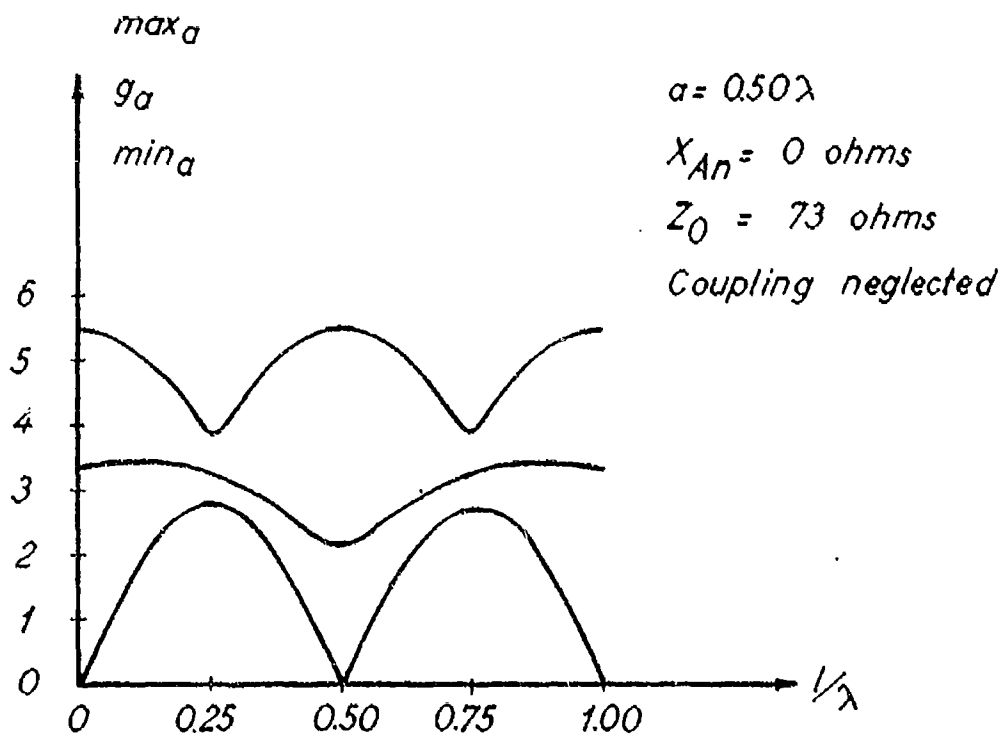


Fig. 25. \max_a , g_a , and \min_a as a function of the length of the lines.

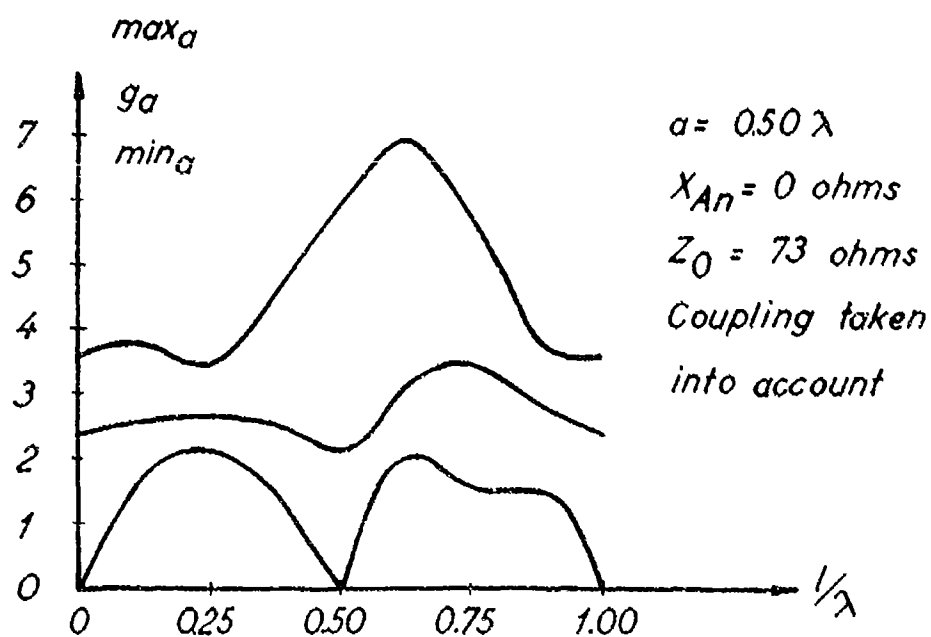


Fig. 26. \max_a , g_a , and \min_a as a function of the length of the lines.

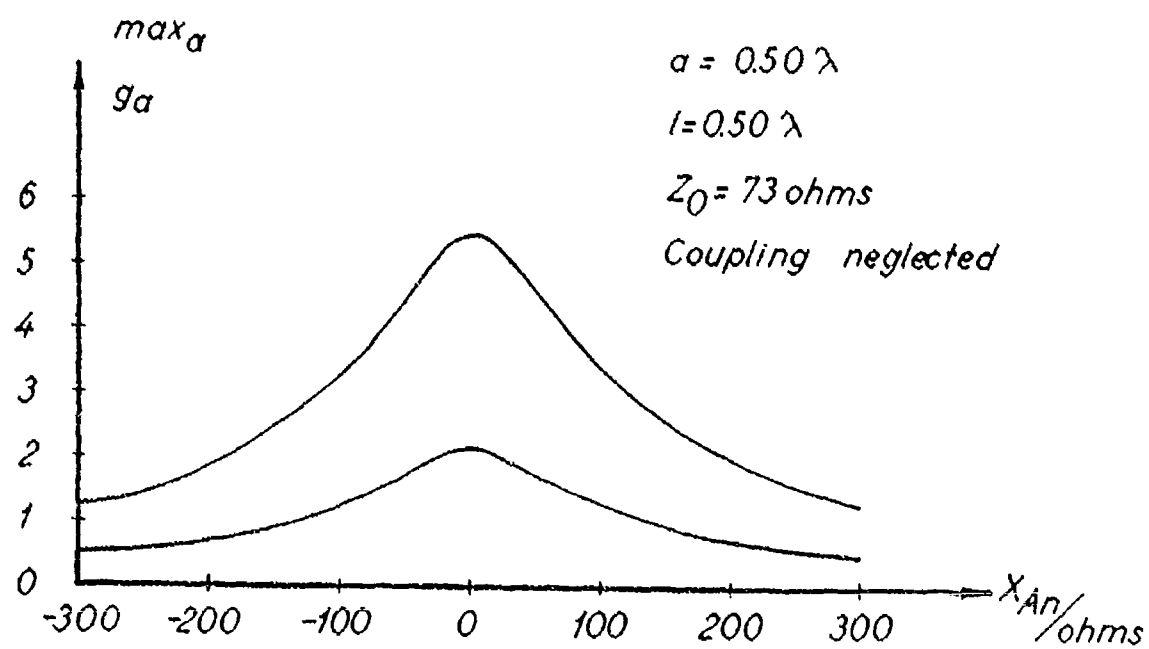


Fig. 27. \max_{α} and g_{α} as a function of X_{An} .

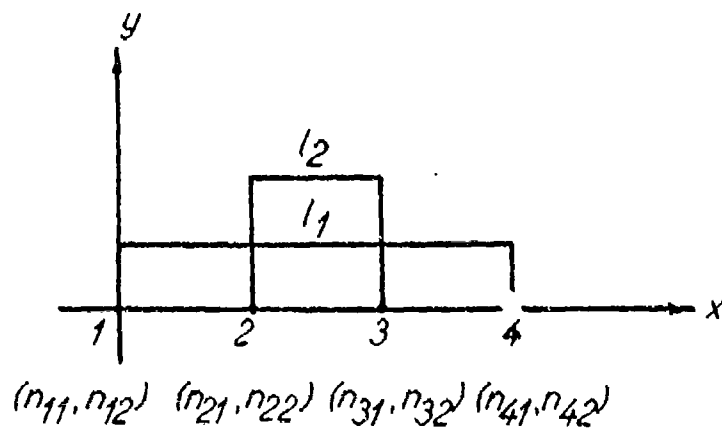


Fig.28. The coordinates of the dipoles.

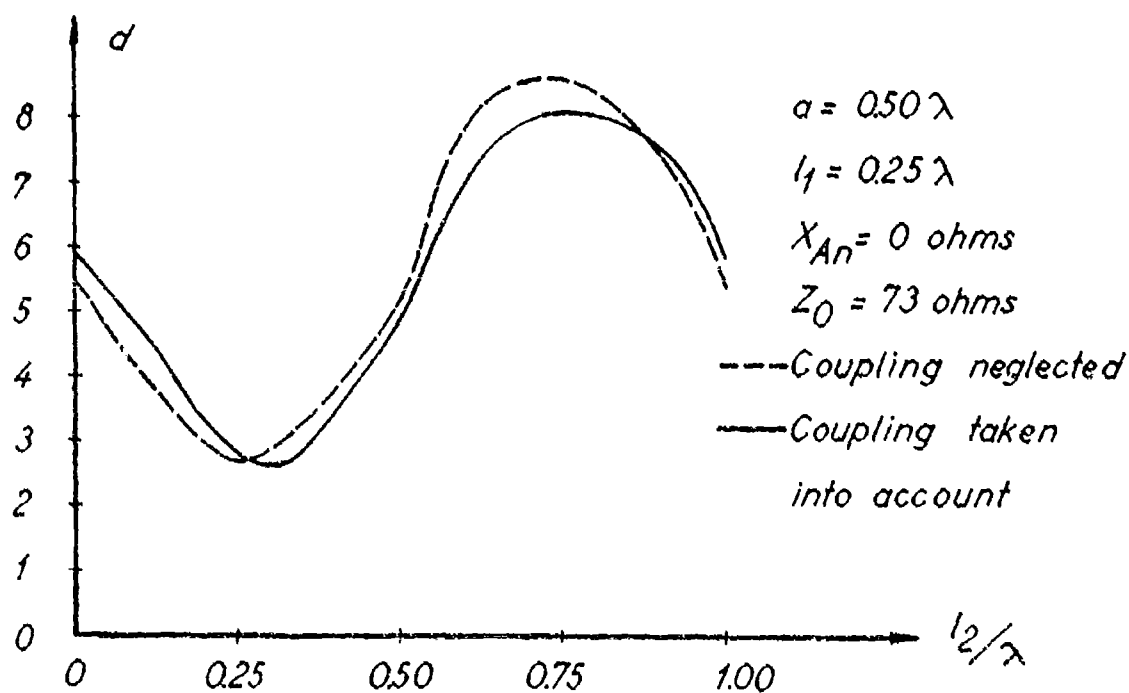


Fig.29. d as a function of l_2 .

$$n_{12} = n_{22} = n_{32} = n_{42} = 0\lambda$$

$$n_{21} = 0.5\lambda, n_{31} = 1.0\lambda, n_{41} = 1.5\lambda$$

$$l_1 = 0.25\lambda$$

$$X_{An} = 0 \text{ ohms}$$

$$Z_0 = 73 \text{ ohms}$$

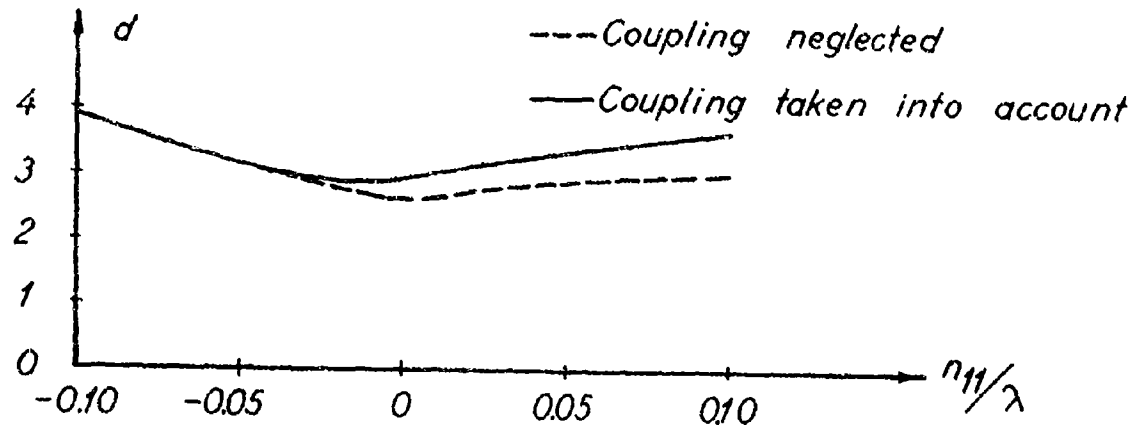


Fig. 30 d as a function of n_{11} .

$$n_{21} = 0.5\lambda, n_{31} = 1.0\lambda, n_{41} = 1.5\lambda$$

$$n_{22} = n_{32} = n_{42} = n_{11} = 0\lambda$$

$$l_1 = 0.25\lambda$$

$$X_{An} = 0 \text{ ohms}$$

$$Z_0 = 73 \text{ ohms}$$

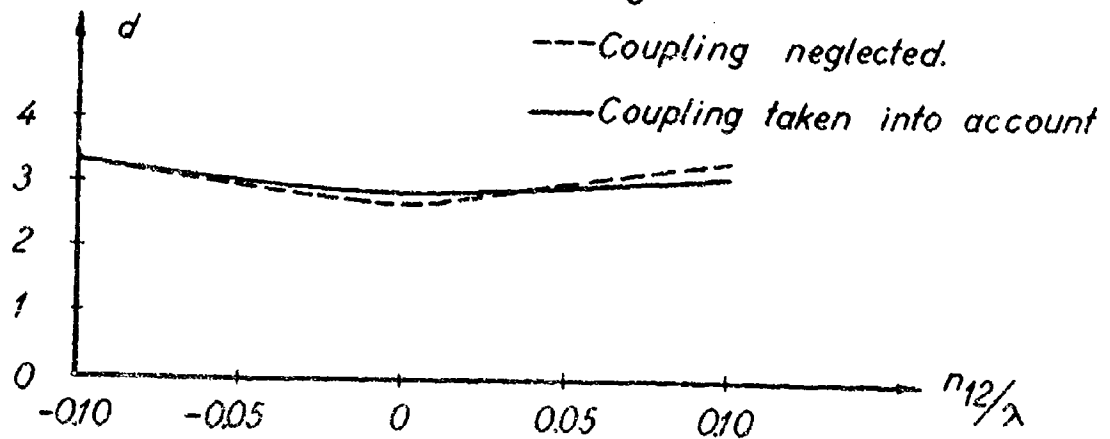


Fig. 31. d as a function of n_{12} .

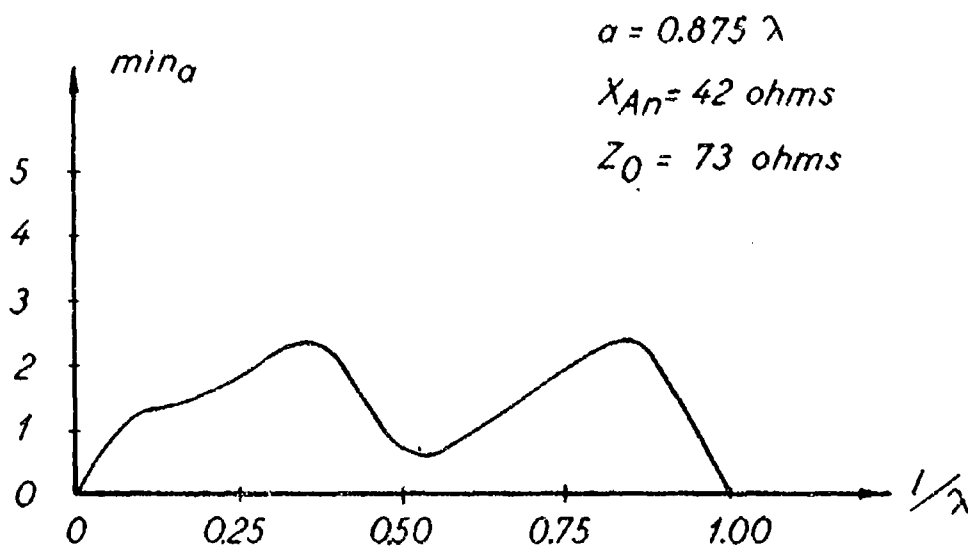


Fig.32. Characteristic variation of \min_a as a function of the length.
Coupling taken into account.

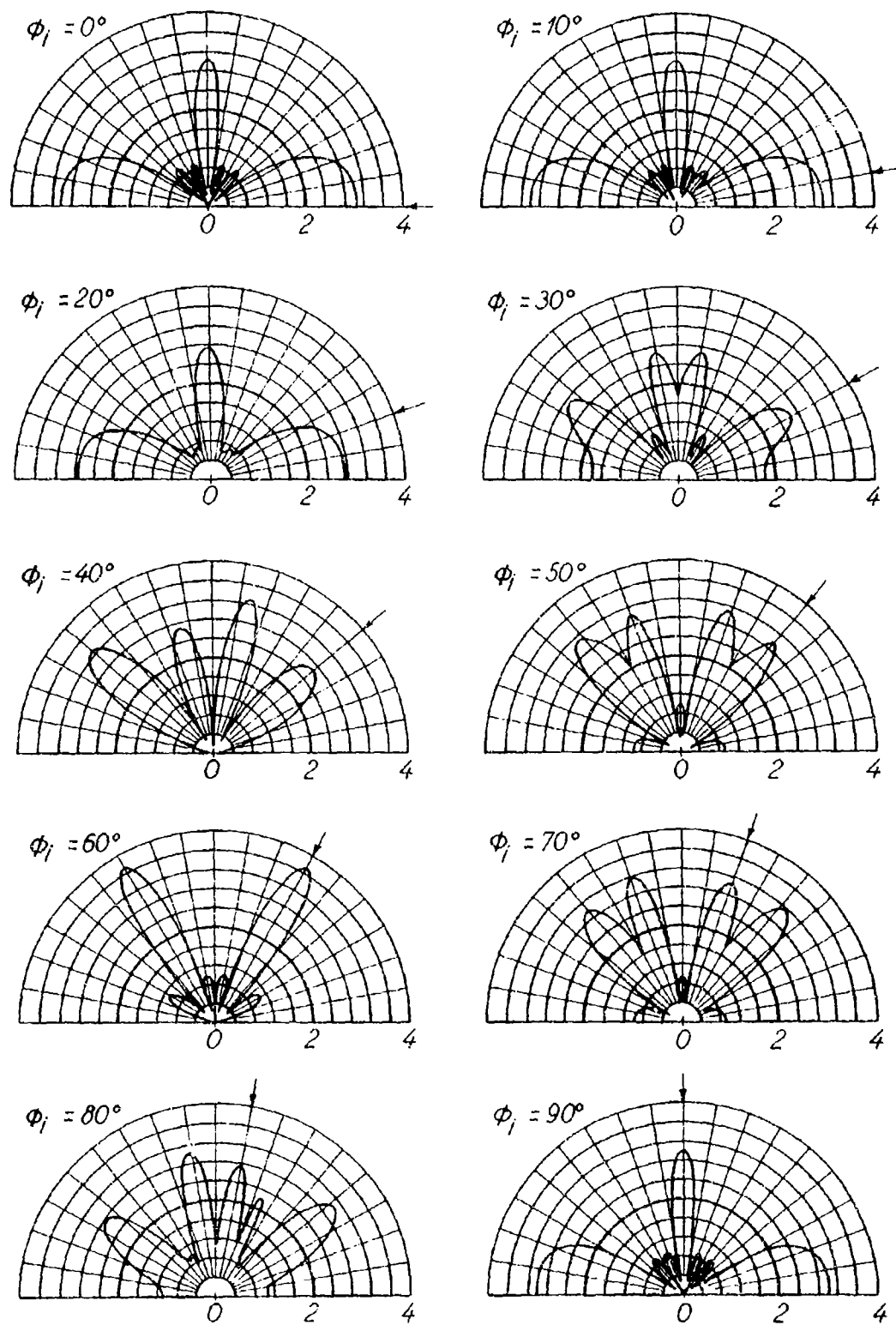


Fig.33. Reradiation patterns for the reflector with the largest value of \min_{α} and d as small as possible when $a \ll 1\lambda$. Coupling is taken into account. For characteristics see Table 3.

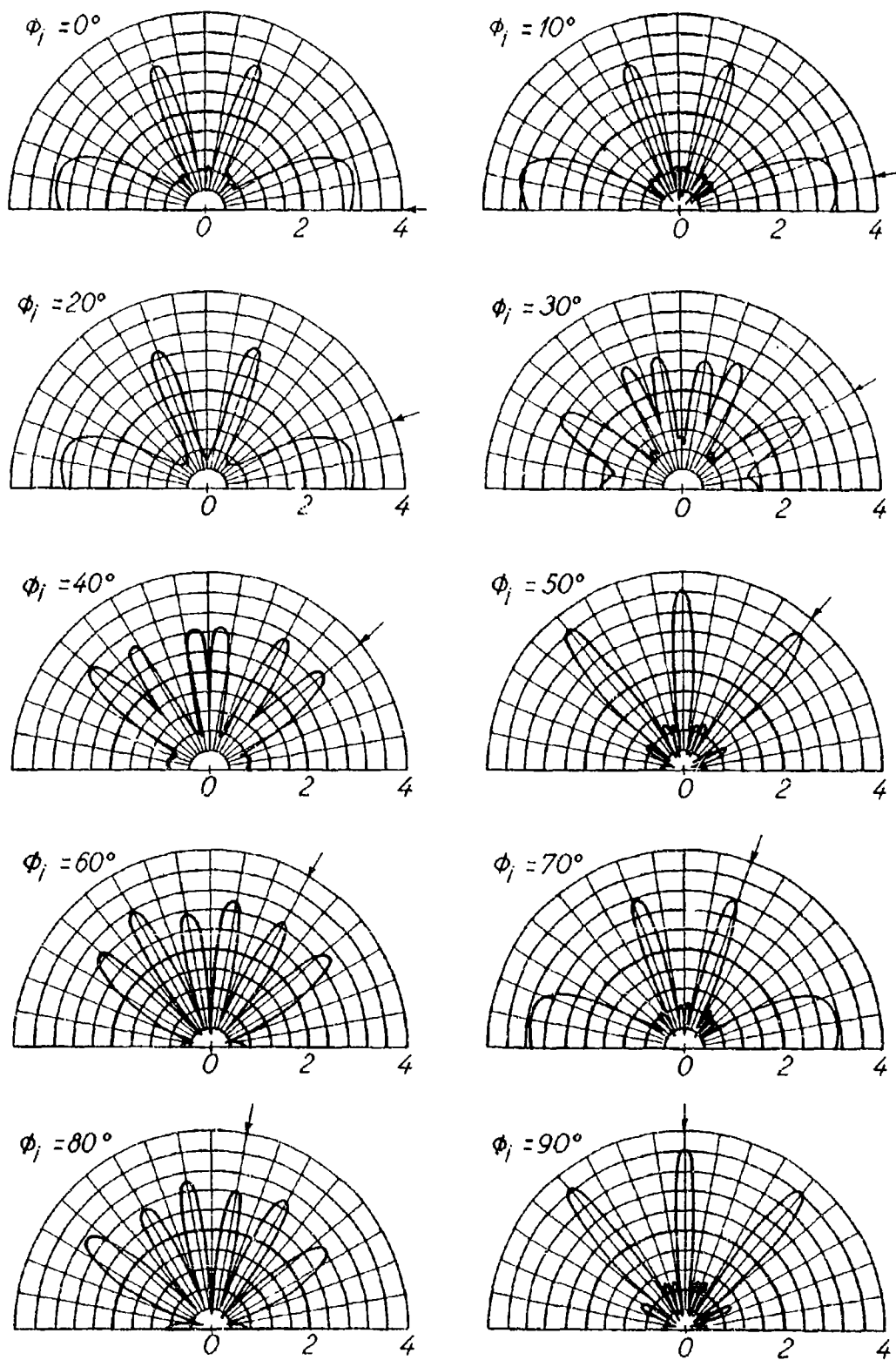


Fig.34. Reradiation patterns for the reflector with the largest value of $\min a_i$, and d as small as possible when $a \leq 3\lambda$. Coupling is taken into account. For characteristics see Table 3.

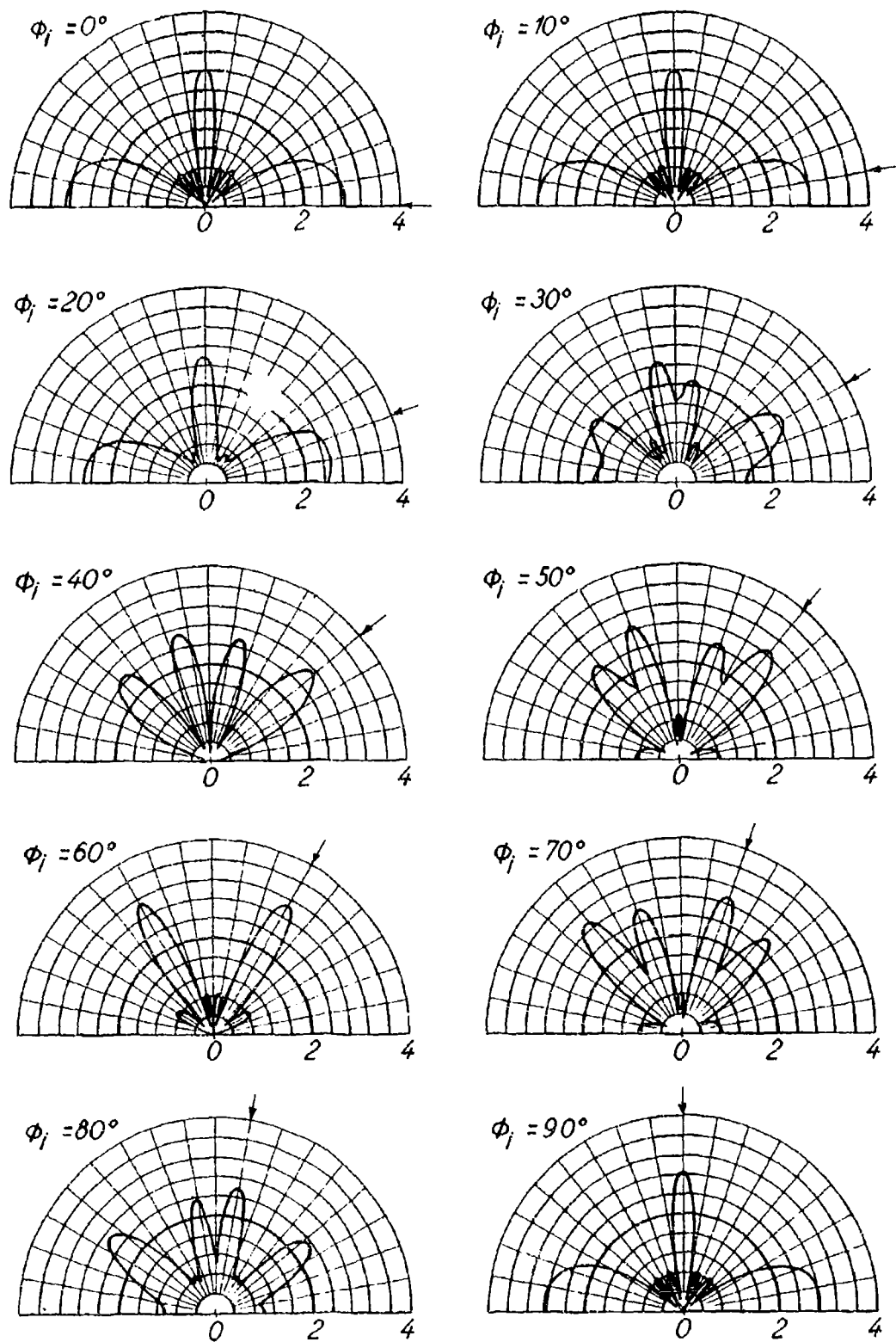


Fig. 35. Reradiation patterns for the reflector with $\min a \geq 25$,
and d as small as possible when $a \leq 1\lambda$.
Coupling is taken into account
For characteristics see Table 3.

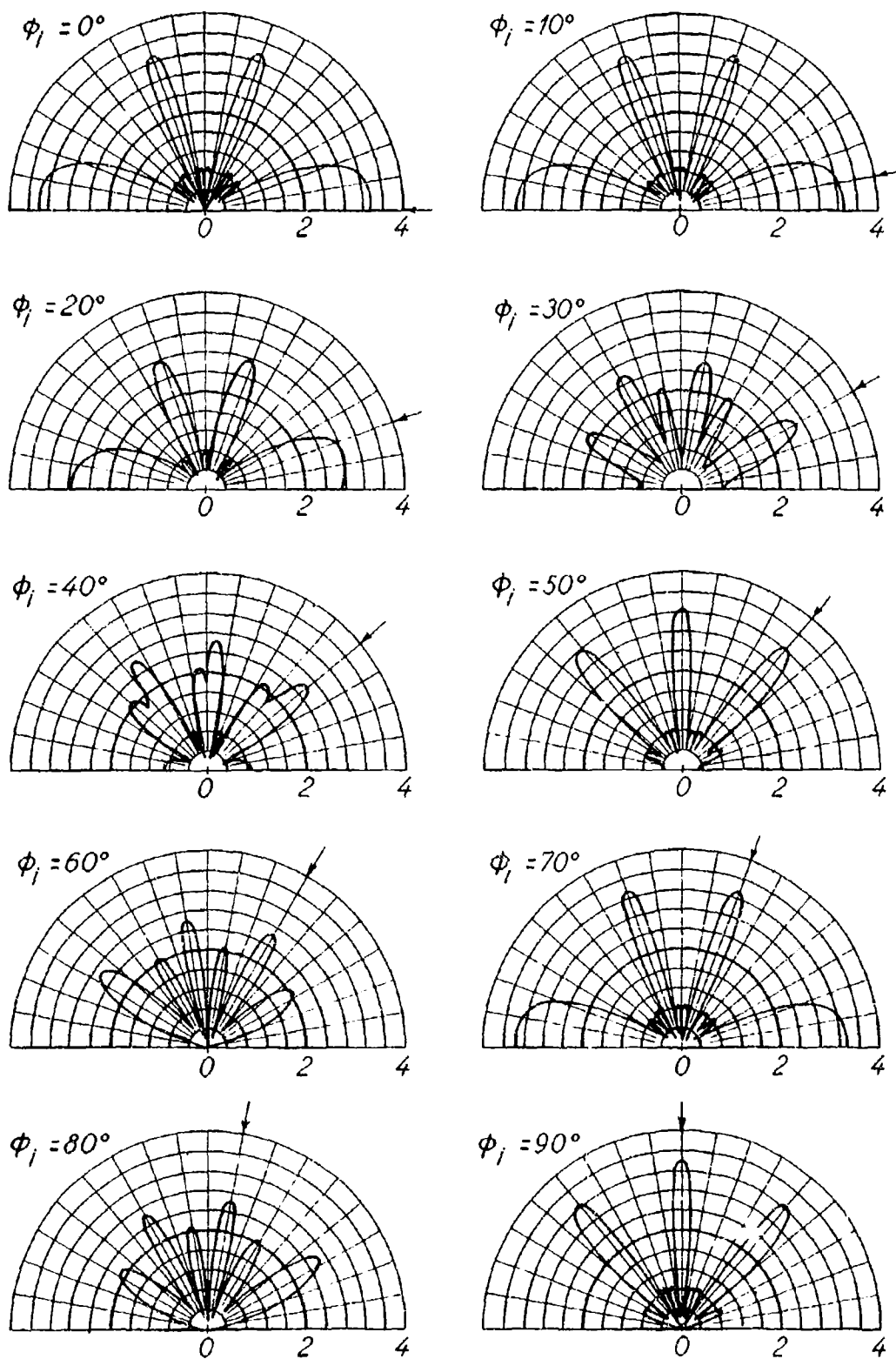


Fig.36. Reradiation patterns for the reflector with $\min_a \geq 2.5$, and d as small as possible when $a \leq 3\lambda$. Coupling is neglected. For characteristics see Table 3.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D		
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13. ABSTRACT An optimization of the reradiated field from a linear Van Atta reflector consisting of four equispaced parallel half-wave dipoles is made. Before the optimization process is discussed, a detailed theoretical investigation is made. A general expression for the reradiated field is established so that it is possible to study the influence of asymmetries in the location of the dipoles, unequal length of the transmission lines, and a mismatch between the dipoles and the transmission lines. In particular, the dependence of the reradiation pattern on the length and the characteristic impedance of the transmission lines, the imaginary part of the dipole impedances, and the spacing between the dipoles, is investigated. The reradiation pattern is optimized so that the back-scattering, as a function of the angle of incidence, has its largest minimum value. The optimization is performed both when coupling is neglected and when it is taken into account. In the optimization consideration is given to the shape of the reradiation pattern near the direction back in the direction of propagation of an incident wave.		

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Van Atta Reflector						
Retrodirective Reflector						
Linear Array						
Adaptive Antenna System						
Dipoles						
Mutual Impedances						
Optimization						

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